Sq and L: Models and Inverse Problems

D.J. Ivers[(1)], **D.E. Winch**[(1)]

[(1)]*School of Mathematics and Statistics F07*
University of Sydney
NSW, Australia 2006
Email: D.Ivers@maths.usyd.edu.au

July 8, 2009

Abstract

Traditionally Sq has been derived from ground-based (non-polar) data. Standard spectral and spherical harmonic analysis can separate current sources internal to the Earth's surface from external current sources. However, it is not possible to isolate the effects of ionospheric current from ground-based vector data alone: the magnetospheric current effects are aliased into ionospheric current effects. Low altitude satellite vector data offers the possibility of separating magnetic effects of magnetospheric and ionospheric currents. Of course, satellite vector data alone cannot simply separate ionospheric current effects from the effects of current induced in the Earth. Surface and satellite data together may separate all three current regions.

We consider the inverse current problem for Sq based on a spherical surface current model, in particular, the total intensity data problem, uniqueness of the solution and a spectral method of solution. If the data is vector and lies on a spherical surface then it may be decomposed by analysis into internal, external and non-potential parts. The internal and external parts lead to inverse vector problems for the sources. We consider the source inverse problem for surface and shell ionospheric current models. If the data does not lie on a spherical surface, analysis is more difficult: field aligned currents associated with a non-potential magnetic field must be allowed for near the polar regions. The difficulty arises since the radial dependence of magnetic fields due to field aligned currents is not as simply prescribed as that of harmonic fields. We outline several ways to model field aligned currents. Similar considerations apply to the weaker lunar variation L.

1 Total Intensity S_q at Satellite Altitude

Fig.1–3 show the total intensity Sq (with contamination by the magnetospheric and induced fields) at equal times from UT20:30 to 04:30, derived from Champ total intensity data from 15 May to 25 September 2001 with Kp $<$ 3+:

- The 93.5 minute orbital period gave about 2037 daytime equatorial crossings during 4.34 standard months reasonably uniformly distributed in longitudes.
- Data was binned on 1 longitude by 1 latitude and one hour of Universal Time.
- The departure of the Champ orbit from circularity was removed using the main field model of Olsen (2002) for epoch 2000.
- A heavy zonal trend in the binned residues for any particular Universal Time, which masked the Sq variation, and the unwanted lithospheric field anomalies, were successfully removed by subtracting the local midnight values.

See [1] for a full discussion. From the magnetic data the current sources can be determined, as shown in the next section, if the current lies in a spherical surface.

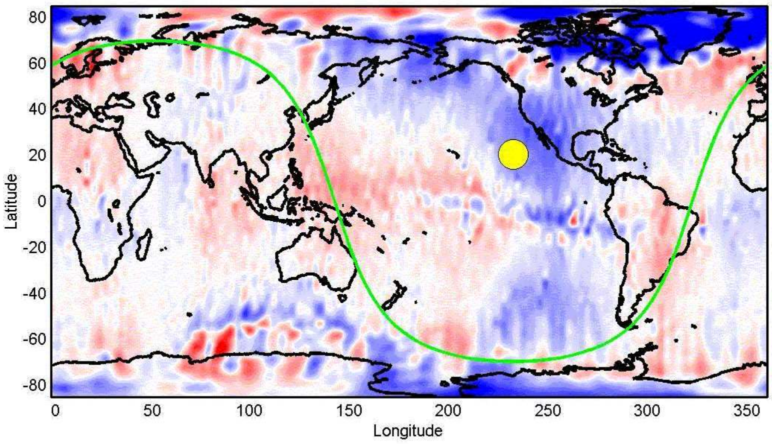

Figure 1: UT20:30.

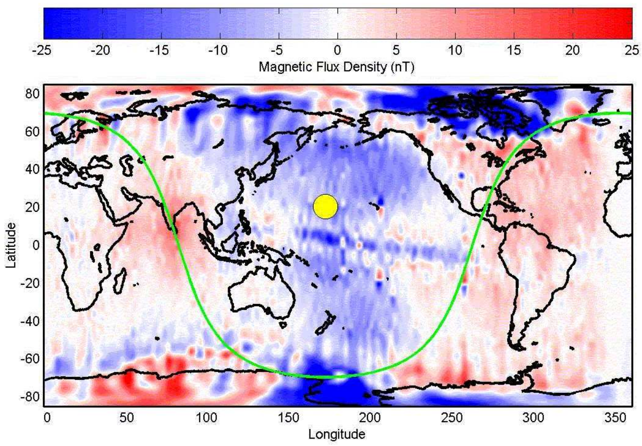

Figure 2: UT00:30.

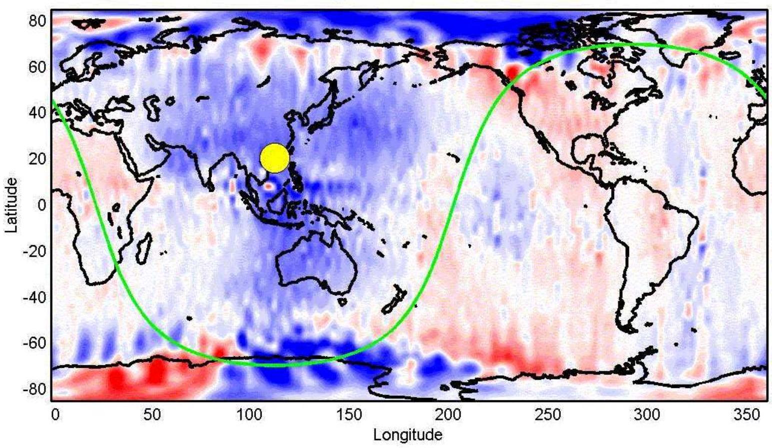

Figure 3: UT04:30.

2 The Total Intensity S_q Inverse Problem

The ionosphere is modelled as a spherical current sheet, i.e. a toroidal current, which is the source of S_q. We neglect the contribution of the magnetospheric and induced currents. Let $\mathbf{B} = \mathbf{B}_{\mathrm{mf}} + \mathbf{B}_{\mathrm{sq}}$. Then

$$\begin{aligned}
\nabla\times \mathbf{B}_{\mathrm{Sq}} &= \mathbf{0}\,, \qquad r < a\,;\\
[\mathbf{B}]_a &= 0\,, \qquad r = a\,;\\
\nabla\times \mathbf{B} &= \mathbf{0}\,, \qquad a < r < b\,;\\
[\mathbf{n}\cdot\mathbf{B}]_b &= 0 \quad [\mathbf{n}\times\mathbf{B}]_b = \mu_0 \mathbf{J}_{\mathrm{Sq}}\,, \qquad r = b\,;\\
\nabla\times \mathbf{B} &= \mathbf{0}\,, \qquad b < r < c\,;\\
|\mathbf{B}| &= D\,, \qquad r = c\,.
\end{aligned}$$

The satellite total intensity data on $r = c$ is D.

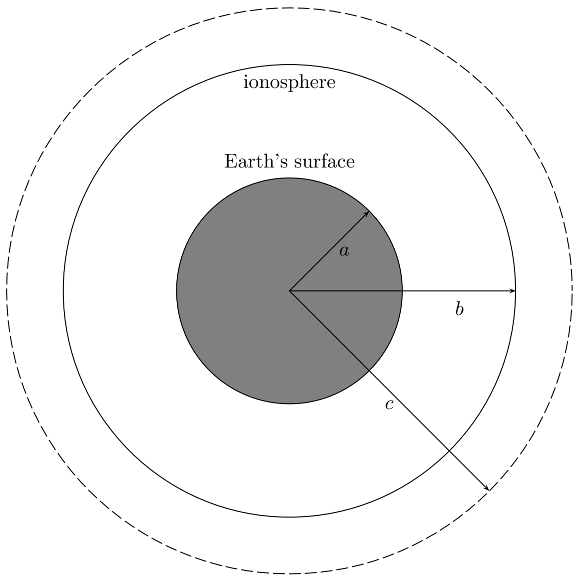

Figure 4: Geometry (not to scale) of the problem.

The problem is non-linear in the magnetic field, but linear (to first order) in the magnetic variations $\mathbf{B}_{\mathrm{Sq}}$:

$$\begin{aligned}
\nabla\times \mathbf{B}_{\mathrm{Sq}} &= \mathbf{0}\,, \qquad r < b\,;\\
[\mathbf{n}\cdot\mathbf{B}_{\mathrm{Sq}}]_b &= 0 \quad [\mathbf{n}\times\mathbf{B}_{\mathrm{Sq}}]_b = \mu_0 \mathbf{J}_{\mathrm{Sq}}\,, \qquad r = b\,;\\
\nabla\times \mathbf{B}_{\mathrm{Sq}} &= \mathbf{0}\,, \qquad r > b\,;\\
\widehat{\mathbf{B}}_{\mathrm{mf}}\cdot\mathbf{B}_{\mathrm{Sq}} &= d\,, \qquad r = c\,.
\end{aligned}$$

The satellite total intensity data given on $r = c$ is d. Let $\mathbf{B}_{\mathrm{sq}} = -\nabla V_{\mathrm{sq}}$. Expand in spherical harmonics

$$V^m_{Sq,n}(r) = \begin{cases} V^m_{Sq,n}(b-)\left(\dfrac{r}{b}\right)^n, & \text{in } r < b;\\ V^m_{Sq,n}(b+)\left(\dfrac{b}{r}\right)^{n+1}, & \text{in } r > b. \end{cases}$$

Note the solution holds in $r < a$. The jump condition on the normal field across the ionospheric layer implies $nV^m_{Sq,n}(b-) = -(n+1)V^m_{Sq,n}(b+)$. The other condition implies $\mu_0\mathbf{J}_{\mathrm{Sq}} = \nabla[V_{\mathrm{sq}}]_b \times \mathbf{1}_r$, i.e. the ionospheric current sheet is toroidal, $\mu_0\mathbf{J}_{\mathrm{Sq}} = \nabla\times(T_{\mathrm{sq}}\mathbf{r})$, with potential $bT_{\mathrm{sq}} = [V_{\mathrm{sq}}]_b$.

The data gives $d = -\widehat{\mathbf{B}}_{\mathrm{mf}}\cdot\nabla V_{\mathrm{sq}}$ at $r = c$. Thus, expanding the main field in vector spherical harmonics,

$$dB_{\mathrm{mf}} = -\mathbf{B}_{\mathrm{mf}}\cdot\nabla V_{\mathrm{sq}} = -\sum_{\alpha,\beta,\gamma} B^{\alpha}_{\mathrm{mf}} G(\beta,\beta_1)\langle \mathbf{Y}_\alpha\cdot\mathbf{Y}_\beta, Y_\gamma\rangle V^{\beta}_{\mathrm{sq}}(c) Y_\gamma\,,$$

where the angle brackets denote the inner-product on the unit sphere. These equations constitute the inverse problem for V_{sq} and hence $\mathbf{B}_{\mathrm{Sq}}$. $\mathbf{Y}_\alpha$, etc denote vector spherical harmonics (see [2]).

3 The Vector Sq Problem

Only vector data allows the possibility of separating current sources internal and external to the surface on which the magnetic field measurements are made. We relate the magnetic multipole measurements and then examine their implications for the unique determination of the sources.

The magnetic vector potential $\mathbf{A}$ is given in terms of the current density $\mathbf{J}$ in the region Ω by

$$\mathbf{A}(\mathbf{r}') = \frac{\mu_0}{4\pi}\int_{r<a\cup r>b} \frac{\mathbf{J}}{|\mathbf{r}-\mathbf{r}'|}\, dV\,. \tag{1}$$

If $a<r<b$ is insulating the magnetic vector potential can be expanded in a descending and ascending powers of r and vector moments of the current density in $r<a$ and $r>b$. For $r>r'$ the following identity is true,

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{r}\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\frac{4\pi}{2n+1}\left(\frac{r'}{r}\right)^n Y_n^{m\,*}(\theta,\phi)Y_n^m(\theta',\phi')\,.$$

Thus in $a<r<b$ $\mathbf{A}=\mathbf{A}_{r<a}+\mathbf{A}_{r>b}$, where

$$\mathbf{A}_{r<a} = \mu_0\sum_{n=1}^{\infty}\frac{1}{2n+1}\sum_{m=-n}^{n}\left\{\int_{r<a}\mathbf{J}'Y_n^{m\,*}(\theta',\phi')(r')^n\, dV'\right\}\frac{Y_n^m(\theta,\phi)}{r^{n+1}}\,.$$

$$\mathbf{A}_{r>b} = \mu_0\sum_{n=1}^{\infty}\frac{1}{2n+1}\sum_{m=-n}^{n}\left\{\int_{r>b}\frac{\mathbf{J}'Y_n^{m\,*}(\theta',\phi')}{(r')^{n+1}}\, dV'\right\}Y_n^m(\theta,\phi)r^n\,.$$

In the current-free region $a<r<b$, $\mathbf{B}=-\nabla V$ where $\nabla^2 V=0$. The solution has the representation

$$V = a\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left\{M_n^m(t)\left(\frac{r}{a}\right)^{-(n+1)} + N_n^m(t)\left(\frac{r}{a}\right)^n\right\}Y_n^m(\theta,\phi)\,.$$

The multipole moments and Gauss coefficients are related by

$$g_n^m - ih_n^m = \sqrt{\frac{(2n+1)(2-\delta_0^m)}{4\pi}}M_n^m\,, \qquad q_n^m - is_n^m = \sqrt{\frac{(2n+1)(2-\delta_0^m)}{4\pi}}N_n^m\,.$$

V can also be expressed in terms of the current. Integrating from a reference point $\mathbf{r}_0$, where the magnetic potential $V=0$, to $\mathbf{r}$ over any path C not intersecting the conducting region Ω,

$$\begin{aligned} V(\mathbf{r}) &= \int_C \nabla V\cdot d\mathbf{l} = -\int_C \nabla\times\mathbf{A}\cdot d\mathbf{l} \\ &= -\frac{\mu_0}{4\pi}\int_V\int_C \nabla\frac{1}{|\mathbf{r}-\mathbf{r}'|}\times\mathbf{J}'\cdot d\mathbf{l}\, dV'\,. \end{aligned}$$

This expression is difficult to evaluate. Let $V=V_{r<a}+V_{r>b}$. If every point in $r<a$ can be joined to $\mathbf{r}_0=\infty$ by a radial path, then $d\mathbf{l}=\mathbf{1}_r dr$ and the magnetic scalar potential due to currents in $r<a$ is given by

$$V_{r<a} = \frac{\mu_0}{4\pi}\int_{r=a} G_i(\mathbf{r};\mathbf{r}')\mathbf{1}_{r'}\times\mathbf{J}'\cdot d\mathbf{S} + \frac{\mu_0}{4\pi}\int_{r<a} G_i(\mathbf{r};\mathbf{r}')\mathbf{1}_{r'}\cdot\nabla'\times\mathbf{J}'dV'$$

where

$$G_i(\mathbf{r};\mathbf{r}') := \int_r^{\infty}\frac{r'\, dr}{r|\mathbf{r}-\mathbf{r}'|} = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\frac{4\pi}{(2n+1)(n+1)}\left(\frac{r'}{r}\right)^{n+1} Y_n^{m\,*}(\theta,\phi)Y_n^m(\theta',\phi')\,.$$

The surface integral vanishes since $r=a$ is a sphere. Thus

$$V_{r<a} = \frac{\mu_0}{4\pi}\int_{r<a} G_i(\mathbf{r};\mathbf{r}')\mathbf{1}_{r'}\cdot\nabla'\times\mathbf{J}'dV'\,.$$

In the degenerate case, where the spherical shell becomes a spherical surface of radius a, $\mathbf{J}=\mathbf{J}_S\delta(r-a)$. Then

$$V_{r<a} = \frac{\mu_0}{4\pi a}\int_{r=a} G_i(\mathbf{r};\mathbf{r}')\boldsymbol{\Lambda}'\cdot\mathbf{J}_S' dS'\,.$$

If every point in $r>b$ can be joined to the origin $P_0=O$ by a radial path. Proceeding as in the previous case, the magnetic scalar potential due to currents within the volume V_e is given by

$$V_{r>b} = \frac{\mu_0}{4\pi}\int_{r=b} G_e(\mathbf{r};\mathbf{r}')\mathbf{1}_{r'}\times\mathbf{J}'\cdot d\mathbf{S}' + \frac{\mu_0}{4\pi}\int_{r>b} G_e(\mathbf{r};\mathbf{r}')\mathbf{1}_{r'}\cdot\nabla'\times\mathbf{J}'dV'$$

where

$$G_e(\mathbf{r};\mathbf{r}') := -\int_0^r \left(\frac{r'}{r|\mathbf{r}-\mathbf{r}'|}-1\right) dr = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} 4\pi n(2n+1)\left(\frac{r}{r'}\right)^n Y_n^{m\,*}(\theta,\phi)Y_n^m(\theta',\phi'),$$

and $f(r') = 1/r'$ has been chosen to make the integral for G_e finite. Thus the magnetic scalar potential due to currents in $r > b$ can be expanded in ascending powers of r in $r < b$. The surface integral vanishes if ∂V is a spherical shell,

$$V_{r>b} = \frac{\mu_0}{4\pi}\int_{r>b} G_e(\mathbf{r};\mathbf{r}')\mathbf{1}_{r'}\cdot\nabla'\times\mathbf{J}'dV'.$$

In the degenerate case, where the spherical shell becomes a spherical surface of radius a, $\mathbf{J} = \mathbf{J}_S\delta(r-a)$. Then

$$V_{r>b} = \frac{\mu_0}{4\pi a}\int_{r=b} G_e(\mathbf{r};\mathbf{r}')\boldsymbol{\Lambda}'\cdot\mathbf{J}_S' dS'.$$

A similar analysis leads to analogous expressions for the multipole moments derived from vector satellite data in terms of the current sources in $r < c$ and $r > c$. Combining these with the previous results leads to expressions relating current sources in $r < a$, $b < r < c$ and $r > c$ to the two sets of multipole moments.

3.1 Significance of the Magnetic Multipole Moments

The magnetic multipole moments are related to the electric currents inside the conducting region. From the above results it follows that

$$M_n^m = \frac{\mu_0}{(n+1)(2n+1)a^{n+2}}\int_{r<a} r^n Y_n^{m*}\mathbf{r}\cdot\nabla\times\mathbf{J}\,dV,$$

$$N_n^m = \mu_0(n+1)(2n+1)a^{n-1}\int_{r>b}\frac{Y_n^{m*}\mathbf{r}\cdot\nabla\times\mathbf{J}}{r^{n+1}}dV.$$

The extent to which the multipole moments determine the electric current and the magnetic Let $\mu_0\mathbf{J} = \mathbf{T}\{T\} + \mathbf{S}\{S\}$ be the toroidal-poloidal representation. Then

$$M_n^m = \frac{n}{(2n+1)a^{n+2}}\int_0^a r^{n+2}T_n^m(r,t)\,dr.$$

The multipole moments provide only very limited information about the current distribution inside $r < a$. As n increases the weight $(r/a)^n$ limits this information to the part of the conducting region in $r < a$ closer and closer to the boundary $r = a$. M_n^m provides only one of the moments of $T_n^m(r,t)$, whereas all the moments

$$\int_0^a r^k T_n^m(r,t)\,dr, \qquad k = 0,1,2,\ldots,$$

are required to uniquely determine $T_n^m(r,t)$. In the degenerate case of a spherical surface

$$M_n^m = \frac{n}{(2n+1)}T_n^m.$$

Similarly for N_n^m.

By imposing further arbitrary conditions we now determine a current, which is consistent with the limited information available from the multipole moments. In terms of the inner-product,

$$(f,g) = \int_0^a f(r)^* g(r) r^2\,dr,$$

of functions of r,

$$(r^n, T_n^m) = \frac{2n+1}{n}a^{n+2}M_n^m,$$

which shows that M_n^m determines "the component of T_n^m along r^n". The solution for $\mu_0\mathbf{J}$ is completed by decomposing T_n^m into orthogonal components,

$$T_n^m = T_{n\perp}^m + T_{n\|}^m, \qquad (T_{n\perp}^m, T_{n\|}^m) = 0,$$

$$T_{n\perp}^m = T_n^m - \frac{(r^n, T_n^m)}{(r^n, r^n)}r^n$$

and

$$T_{n\|}^m = \frac{(r^n, T_n^m)}{(r^n, r^n)} r^n = \frac{(2n+1)(2n+3)}{na} \left(\frac{r}{a}\right)^n M_n^m .$$

There is an orthogonal decomposition of T,

$$T = T_\perp + T_\| , \qquad \int_V T_\perp T_\| \, dV = 0 .$$

Hence

$$\mu_0 \mathbf{J} = \mathbf{T}\{T_\|\} + \mathbf{T}\{T_\perp\} + \mathbf{S}\{S\} .$$

Since only a part of the toroidal current is determined by the external multipole moments, the other part of the toroidal current and the poloidal current remaining undetermined, the external multipole moments provide information about only a part of the poloidal magnetic field. The remaining part of the poloidal field and the toroidal magnetic field cannot be determined from the multipole moments. If T_B and S_B are the toroidal and poloidal potentials of the magnetic field, respectively, then $T_B = S$ and $S_B = S_\| + S_\perp$, where $\nabla^2 S_\| = -T_\|$ and $\nabla^2 S_\perp = -T_\perp$. Hence

$$S_\| = \sum_{n=1}^{\infty} \sum_{m=-n}^{m=n} \frac{a M_n^m}{2n} \left\{ (2n+3) \left(\frac{r}{a}\right)^n - (2n+1) \left(\frac{r}{a}\right)^{n+2} \right\} Y_n^m(\theta, \phi) .$$

The magnetic field

$$\mathbf{B} = \begin{cases} \mathbf{S}\{S_\|\} & \text{in } r < a, \\ -\nabla V & \text{in } r > a. \end{cases}$$

The magnetic field associated with $S_\perp$, which satisfies $S_{n\perp}^m = 0$ and $\partial_r S_{n\perp}^m = 0$ at $r = a$, and T_B has zero external magnetic moments,

$$\mathbf{B} = \begin{cases} \mathbf{S}\{S_\perp\} + \mathbf{T}\{T\} & \text{in } r < a, \\ \mathbf{0} & \text{in } r > a. \end{cases}$$

The current of minimum ohmic dissipation with the given multipole moments is

$$\begin{aligned} \int_V (\mu_0 \mathbf{J})^2 \, dV &= \int_V (\mathbf{T}\{T\})^2 \, dV + \int_V (\mathbf{S}\{S\})^2 \, dV \\ &\geq - \int_V T L^2 T \, dV \\ &= \sum_{n=1}^{\infty} n(n+1) \sum_{m=-n}^{n} \left\{ (T_{n\|}^m, T_{n\|}^m) + (T_{n\perp}^m, T_{n\perp}^m) \right\} . \end{aligned}$$

The lower bound on the ohmic dissipation in the conducting region $r < a$ is

$$\int_{r<a} \frac{\mathbf{J}^2}{\sigma} \, dV \geq \frac{a}{\sigma} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \frac{(n+1)(2n+1)^2(2n+3)}{n} \left(\frac{|M_n^m|}{\mu_0} \right)^2 .$$

Analogous results hold for $r > b$, $r < c$, $r > c$. In particular, these provide a lower bound on ohmic dissipation in the ionosphere.

Clearly, vector magnetic measurements on the Earth's surface and at low orbit satellite level may separate ionospheric current sources from magnetospheric and internal sources. It is also clear that the current in a surface model (or integrated layer model) can be uniquely determined from vector magnetic data, but that this is hopelessly false for a (non-zero thickness) shell current model.

4 Electric Field — Uniqueness

Swarm offers the possibility of electric field data. If the electric field is assumed to be constant along magnetic field lines then it can be continued down to the top of the ionosphere. In principle, magnetic and electric field data may lead to uniqueness of the magnetic field.

Assume the induction equation holds in the ionosphere,

$$\partial_t \mathbf{B} = -\nabla \times (\eta \nabla \times \mathbf{B}) + \nabla \times (\mathbf{v} \times \mathbf{B}) .$$

Assume that $\mathbf{B}$ is analytic in $r >= b$ and $\mathbf{B} = \mathbf{0}$ in $r < b$. Using the equations to determine the derivatives of $\mathbf{B}$ at $r = b$ it is found that progress is impossible without knowledge of the normal derivative of the tangential magnetic field. Thus, it does not follow that $\mathbf{B}$ must vanish in $r > b$, if $\mathbf{B}$ vanishes in $r < b$. Thus there may be no uniqueness.

Now suppose that the electric field also vanishes in $r < b$. Continuity of the tangential electric field across $r = b$ leads to $\mathbf{B} = \mathbf{0}$ in $r > b$. Thus $\mathbf{B}$ is unique in $r > b$ if $\mathbf{B}$, $\mathbf{E}$ are given in $r < b$. Measurements of $\mathbf{E}$ at the satellite orbit should provide additional information about $\mathbf{J}_{Sq}$. Extension to anisotropic conductivity also works.

5 Field Aligned Currents

A serious difficulty with satellite data is the presence of field aligned currents, especially near the poles. The magnetic field produced by these currents can be determined from the $\mathbf{Y}_{n,n}^{m}$ vector spherical harmonic (non-potential) coefficients of the magnetic field determined on the spherical surface $r = c$ from vector satellite data. The other vector spherical harmonic coefficients determine the internal and external potential fields [3].

To see this consider field aligned currents modeled by

$$\mu_0 \mathbf{J} = h\mathbf{B}\,.$$

Let

$$\mathbf{B} = \mathbf{B}_{\mathrm{mf}} + \mathbf{B}_1 + \ldots\,, \qquad h = h_0 + h_1 + \ldots$$

where $\mathbf{B}_{\mathrm{mf}}$ is the potential main field. Thus taking the curl,

$$\mu_0 \mathbf{J} = \nabla\times \mathbf{B} = \nabla\times \mathbf{B}_1 + \ldots\,.$$

Hence $h_0 = 0$. To first order,

$$\nabla\times \mathbf{B}_1 = h_1 \mathbf{B}_{\mathrm{mf}}\,.$$

Taking the divergence,

$$\mathbf{B}_{\mathrm{mf}} \cdot \nabla h_1 = 0\,.$$

Thus h_1 is constant along the field lines of $\mathbf{B}_{\mathrm{mf}}$. Knowledge of h_1 on a surface $r = c$ then determines h_1 everywhere (in some valid region).

Can h_1 be determined on $r = c$, if $\mathbf{B}_{\mathrm{mf}} + \mathbf{B}_1$, and hence B_1, is known on the surface $r = c$? Drop the subscripts. Then $\nabla\times \mathbf{B} = h\mathbf{B}_{\mathrm{mf}}$. In harmonics,

$$h\mathbf{B}_{\mathrm{mf}} = \sum_{\alpha,\beta,\gamma} h_\alpha B_{\mathrm{mf},\beta} (Y_\alpha \mathbf{Y}_\beta, \mathbf{Y}_\gamma) \mathbf{Y}_\gamma\,.$$

Now $\nabla\times \mathbf{B} = h\mathbf{B}_{\mathrm{mf}}$ implies $hB_{\mathrm{mf},r} = -\Lambda^2 T/r$ and $B_{n,n}^{m} = -i\sqrt{n(n+1)}\,T_n^m$. Thus

$$h = \frac{i \sum \sqrt{n(n+1)} Y_n^m B_{n,n}^m / r}{B_{\mathrm{mf},r}}\,.$$

Thus $B_{n,n}^{m}$ determines h and hence the field aligned currents.

If the satellite measurements do not lie on a spherical surface then this must be corrected for and this requires knowledge of the radial dependence of the field aligned currents. Four approaches are:

1. Ignore any radial dependence and drop satellite data point radially onto reference surface.
2. Correct satellite data point onto reference surface using a centric dipole for $\mathbf{B}_{\mathrm{mf}}$. In a magnetic dipole coordinates $V_d = \cos\Theta g_1^0 a^3/r^2$. Thus $B_r^d = 2\cos\Theta g_1^0 (a/r)^3$ and $B_\Theta^d = \sin\Theta g_1^0 (a/r)^3$. Hence the field lines are given by $r = C_1 \sin^2\Theta$, $\Phi = C_2$.
3. Correct satellite data point onto reference surface using an eccentric dipole approximation for $\mathbf{B}_{\mathrm{mf}}$.
4. Correct with $\mathbf{B}_{\mathrm{mf}}$.

6 Conclusions

1. *Sq* can be determined from total intensity data to first-order: magnetospheric and inductive effects are not eliminated.
2. A surface model for the ionosphere current can be determined (approximately) from total intensity measurements.

3. The ionospheric current can be separated from magnetospheric effects with vector data at low-orbit satellite altitude,

4. Earth surface vector data and satellite vector data are required to separate magnetospheric and inductive effects.

5. A surface model for the ionosphere current can be uniquely determined from Earth surface magnetic vector data and satellite magnetic vector data.

6. A (thick) shell model for the ionosphere current cannot be uniquely determined from Earth surface magnetic vector data and satellite magnetic vector data. A lower bound on the ohmic dissipation can be determined.

7. The magnetic field in the conducting region is unique if the magnetic and electric fields are known on the boundary of the conducting region. Measurements of **E** and **B** may sufficient to determine **B** and **J**.

8. Separation of the magnetic field into internal, external and non-potential fields can be determined on a spherical surface. Field aligned currents complicate the analysis due to their complicated radial dependence.

9. Splitting the magnetic field into internal, external and non-potential fields works for oblate spheroid.

10. A similar analysis is possible for L.

7 References

[1] J.P.R. Turner, D.E. Winch, D.J. Ivers and R.J. Stening, "Regular daily variations in satellite magnetic total intensity data", *Annales Geophysicae* **25**, 2167–2174, 2007.

[2] D.J. Ivers, Phillips and C.G. Phillips, "Scalar and vector spherical harmonic spectral equations of rotating magnetohydrodynamics", *Geophys. J. Int.* **175**, 955–974, 2008.

[3] D.E. Winch, D.J. Ivers, J.P.R. Turner and R.J. Stening, "Geomagnetism and Schmidt quasi-normalization", *Geophys. J. Int.* **160**, 487–504, 2005.