Error Analysis of convection mapping techniques as applied to Swarm EFI measurements

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Swarm EFI measurements of the electric field may be processed using spherical cap harmonic analysis (SCHA) mapping techniques to generate an ionospheric convection pattern. The ability of the SCHA approach to map input observations is evaluated through the comparison of simulated Swarm EFI observations with the output convection determined at the input coordinate locations. Input observations are simulated using measurements from the ground-based Super Dual Auroral Radar Network (SuperDARN) radars. A convection map is determined using standard SuperDARN mapping software and then projected onto a Swarm satellite track. Noise is represented through the addition of a random term sampled from a normal distribution with a mean of 0 m/s and a standard deviation of 60 m/s. The simulated Swarm observations are broken into north/south and east/west components and mapped with a SCHA technique using a statistical model to constrain the map. For a maximum fitting degree index and order of *K=10*, it was found that the SCHA mapping technique contributes additional errors that are negligible compared to expected Swarm EFI measurement errors. The degree of accuracy can be improved by increasing the maximum fitting order and relaxed by decreasing the fitting order.

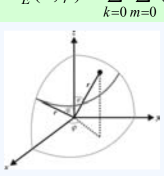
1. Introduction

- Swarm will make continuous observations of the ionospheric plasma drift, making it an ideal instrument for mapping the ionospheric convection pattern
- A spherical cap harmonic analysis (SCHA) technique has been developed for mapping the magnetic field based on observations covering a cap-like region of the spherical Earth (*Haines 1985, 1988*). This method has been adapted for mapping convection based on Swarm EFI observations
- SCHA is very flexible as a multitude of cap-sizes may be used so that both localized and widespread data may be processed to map convection with either open or closed contours at the boundary of the cap

2. Spherical Cap Harmonic Analysis of ionospheric convection

- For a set of observations roughly confined to a spherical cap of angular radius (or cap-size) θ_c, the electrostatic potential is represented by series expansion as

$$\Phi_E(\theta,\varphi) = \sum_{k=0}^{K}\sum_{m=0}^{k}(g_{km}\cos m\varphi + h_{km}\sin m\varphi)P^m_{n_k(m)}(\cos\theta),$$



- Φ_E – electrostatic potential
- θ, φ – magnetic co-latitude, longitude
- P – Associated Legendre Polynomial
- $n_k(m)$ – non-integer degree of fit
- k, m – integer degree index, order of fit
- g_{km}, h_{km} – fitting coefficients
- K – maximum degree index/order of fit

- $n_k(m)$ is determined by confining the Associated Legendre Polynomial at the boundary of the spherical cap using

$$\left.\frac{dP^m_{n_k(m)}(\cos\theta)}{d\theta}\right|_{\theta=\theta_c} = 0, k-m \quad even$$

$$\left.P^m_{n_k(m)}(\cos\theta)\right|_{\theta=\theta_c} = 0, k-m \quad odd$$

- Φ_E is related to velocity by the magnetic field through

$$\overline{E} = -\nabla\Phi_E \qquad \overline{v} = \frac{\overline{E}\times\overline{B}}{B^2}$$

- The fitting coefficients g_{km} and h_{km} are determined by minimizing the difference between the velocity observed by the Swarm EFI and the velocity represented above
- If there is an insufficient distribution of observations to accurately map the entire cap region then a statistical model may be used to constrain the fit

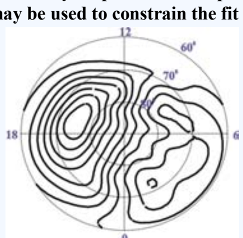


Figure 1: Sample statistical model of ionospheric convection for a maximum degree index and order of *K=10* and a low-latitude flow boundary of 55⁰ for IMF Bz<0 and 6<BT<12 nT, as derived from the ground-based SuperDARN radars (see *Ruohoniemi and Greenwald*, 1996)

- Haines, G. V. , Spherical cap harmonic analysis, *J. Geophys. Res.*, 90(B3):2583-2591, 1985.
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- Ruohoniemi, J. M., and R. A. Greenwald. Statistical patterns of high-latitude convection obtained from Goose Bay HF radar observations. *J. Geophys. Res.*, 101(A10):21742-21763, 1996.
- Ruohoniemi, J. M., and K. B. Baker. Large-scale imaging of high-latitude convection with Super Dual Auroral Radar Network HF radar observations. *J. Geophys. Res.*, 103:20797-20811, 1998.

3. Methodology

- Sample observations of ionospheric plasma drift observed by the Swarm EFI instrument were generated based on an ionospheric convection map determined using observations from the ground-based SuperDARN radars and constrained using the statistical model shown in Figure 1
- This convection was determined using the standard FIT-technique (*Ruohoniemi and Baker*, 1998), and limited to the high-latitude region poleward of 55⁰ magnetic latitude at an altitude of 300 km
- The convection pattern was projected onto a sample Swarm satellite trajectory at a 7.6 km sampling resolution resulting in 931 theoretical 'observations' in a spherical cap of θ_c=30⁰

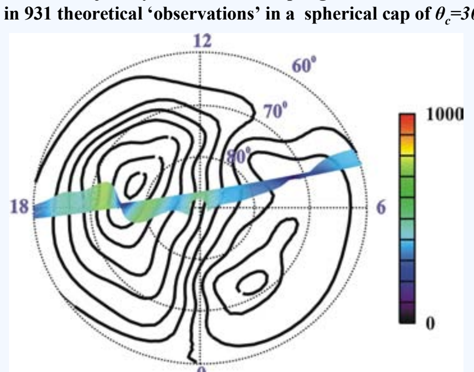
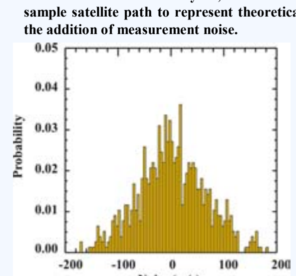


Figure 2: SuperDARN-derived convection pattern for the two-minute interval of 01:20-01:22 UT on January 03, 2001. Coloured vectors are projected onto a sample satellite path to represent theoretical Swarm EFI observations before the addition of measurement noise.



- Simulated Swarm EFI observations are represented by adding normally-distributed noise (see left) to the sampled SuperDARN convection map

Figure 3: Normal distribution with σ=60 m/s

- The simulated Swarm EFI observations were broken into north/south and east/west components and mapped using the SCHA technique using a maximum fitting degree index and order of *K=10*

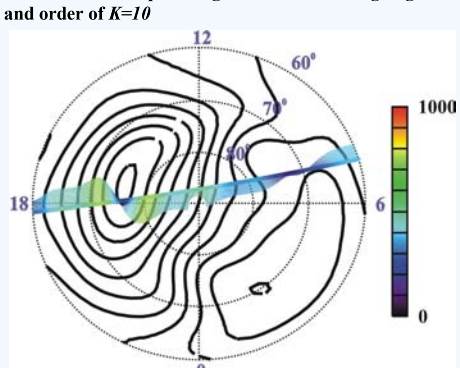


Figure 4: Ionospheric convection pattern calculated using SCHA from the velocity vectors shown in Figure 2 plus additive noise and the model convection pattern shown in Figure 1

4. Accuracy Results

- The ability of the SCHA approach to map the input Swarm EFI observations may be evaluated by comparing the simulated input observations with the output convection determined at the input coordinates
- Figure 5 below shows a scatter plot of the output convection plotted in Figure 4 versus the input observations determined from the theoretical vectors shown in Figure 2 plus the addition of noise

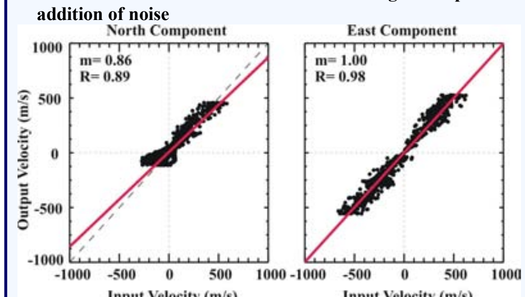


Figure 5: Scatter plot of the input (Figure 2 plus noise) and output (Figure 4) convection separated into north and east components.

- Errors in output data values sampled along the Swarm trajectory agree with input values with errors that are dominated by EFI measurements alone
- In Figure 6, the distribution of the differences between input and output observations is shown

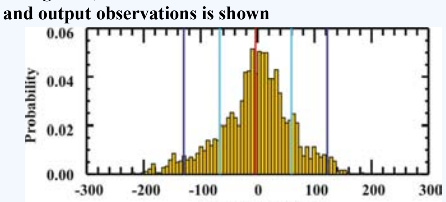


Figure 6: Distribution of the difference between the input and output velocity plotted in Figure 5. The location of the mean and the 1σ and 2 σ cut-offs are indicated by red, blue, and purple lines, respectively.

- The distribution is roughly centered about zero with a mean of -3.60 (red line) and σ=63.03 m/s.
- The degree of accuracy, as measured by the mean and standard deviation of the residual distribution, is dependent on the maximum fitting order used to constrain the fit

K	mean	σ
4	7.64	128.60
6	-2.17	77.25
8	7.66	71.50
10	-3.60	63.03

5. Summary

- Spherical cap harmonic analysis was successfully used to map simulated Swarm EFI observations with constraints from a statistical model
- For a maximum fitting degree index and order of *K=10*, the SCHA mapping technique contributes additional errors that are negligible compared to expected Swarm EFI measurement errors
- In future work the SCHA technique will be applied to real instrument measurements from the DMSP satellite during periods of varying magnetic activity

SuperDARN data provided by the University of Saskatchewan

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