

END-TO-END MISSION SIMULATION STUDY (E2E+)

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INTRODUCTION

End-to-End mission performance simulations are carried out with the aim of analyzing the key system requirements, particularly with respect to Mission design, e.g. the number of Swarm satellites and their orbits, related to the science objectives of Swarm. The Mission Simulator also assists the implementation phases as well as mission operation planning and mission operation.

The Mission Simulations simulate the magnetic measurements of the full Swarm Mission by generation of synthetic data using sophisticated models including magnetic field values for all relevant contributions to Earth's magnetic field: core and lithospheric fields, fields due to currents in the ionosphere and magnetosphere, due to their secondary, induced, currents in the oceans, lithosphere and mantle, and fields due to currents coupling the ionosphere and magnetosphere. In order to be able to use realistic parameters of the Earth's environment, the mission simulation starts in July 1998 and lasts until re-entry of the lower satellites 4.5 years later.

The measurements are subsequently inverted using the Comprehensive Inversion (CI) scheme to analyze various aspects of field recovery in relation to different number of satellites, different constellations and realistic noise sources by comparison of the estimated models with the input models.

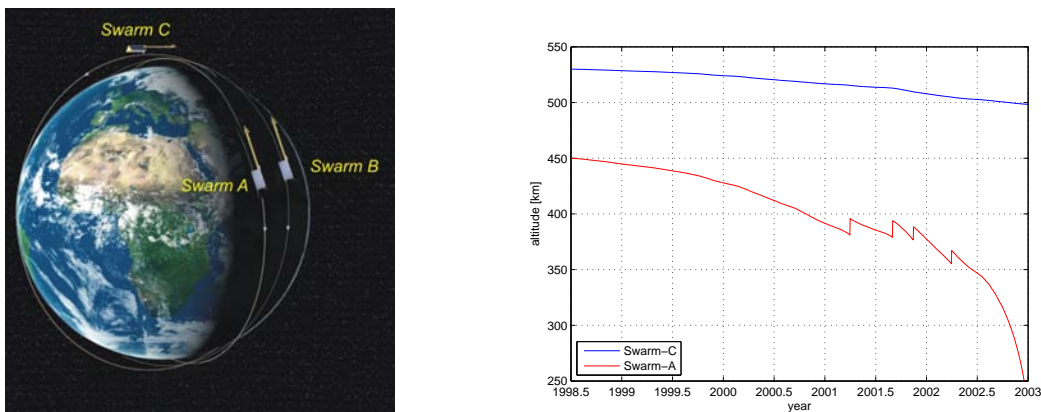
Here we present some results of recent simulations in the framework of extended studies of Mission Simulations, E2E+. The gain of explicitly using gradient information from the lower satellite pair as well as using data from these satellites from the last couple of months just before re-entry is demonstrated. Finally the application of regularization information on the Secular Variation is shown to improve the recovery of this radically.

SWARM CONSTELLATION

The optimal Swarm constellation for the science objectives has been found in previous E2E simulations [1] to consist of three satellites. A lower pair, Swarm A and B, flying side-by-side separated by 1.5° at the Equator injected at 450 km altitude and the upper Swarm C satellite injected at 530 km altitude, see Fig. 1.

The simulation starts on July 1, 1998 – one solar cycle before anticipated launch of Swarm. The simulation length is 4.5 years.

Fig. 1: Swarm Constellation



GENERATION OF SYNTHETIC DATA

The generation of the synthetic data comprises the following sources with colour coding corresponding to Fig. 2.

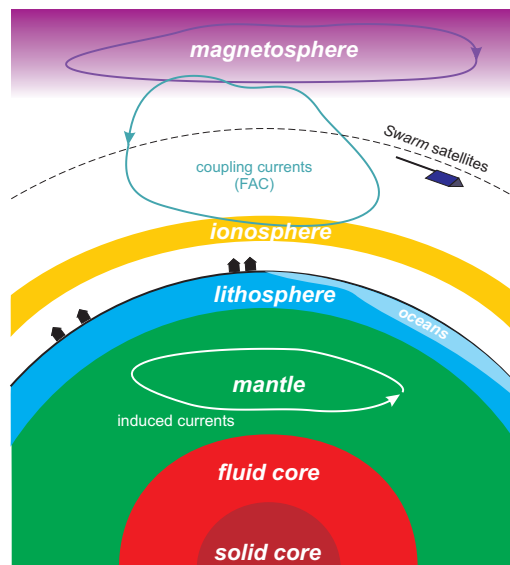
- **Core field time changes** up to $n = 19$
- **Lithospheric field** (static) up to $n = 250$
- **Ionospheric field**, daily + seasonal periodicity. Amplitude modulated by daily values of F10.7. Induced field by means of 1D conductivity model
- **Magnetospheric field** Time dependence of external coefficients up to $n = 3, m = 1$ given by observatory data (hourly mean values)
- **Induced field** (up to $n = 45$) calculated using 3D mantle conductivity, including oceans
- **Toroidal** (non-potential) field daily + seasonal periodicity
- Instrument noise

COMPREHENSIVE INVERSION

The Comprehensive Inversion (CI) uses the approach of *simultaneous* estimation of the sources and structures below. Note: the degrees and orders of the various sources are not identical to the input model of the preceding section.

- **Core field** up to $n = 20$, temporal resolution cubic B-spline, 1 internal knot
- **Lithospheric field** up to $n = 150$
- **Ionospheric field**, quasi-dipole $n = 45, m = 6$, semi-annual and quarter-daily periodicity
- **Toroidal field**, same parametrization as ionospheric field
- **Magnetospheric field** up to $n = 3, m = 1$, 1 hour bins
- **Induced field** up to $n = 3, m = 3$, 1 hour bins
- Instrument alignment parameters (Euler angles), 30 day bins

Fig. 2: Sources for Synthetic Data Generation



$[\nabla\mathbf{B}]_{\text{EW}}$ AND "SELECTIVE INFINITE VARIANCE WEIGHTING"

For the CI the availability of the gradient information, $[\nabla\mathbf{B}]_{\text{EW}}$, is taken explicitly into account to resolve the lithospheric field by use of the so-called "Selective Infinite Variance Weighting" (SIVW) method. Formally data from Swarm A and B are transformed to sums and differences. The high order crustal field is purely resolved by the differences whereas the remaining parameters are resolved by all data (sums, differences and Swarm C) – although the differences contribute very little to resolve e.g. low order core field.

Formally, the equation to solve using this approach is, where **light brown** terms are purely related to the differences, **magenta** terms are related to sums and Swarm C, and **black** terms relate to all data:

$$\begin{pmatrix} \mathbf{d}_+ \\ \mathbf{d}_- \\ \mathbf{d}_C \end{pmatrix} = \begin{pmatrix} G_+ & G_+^\ell & 0 & G_+^h \\ G_- & G_-^\ell & G_-^h & 0 \\ G_C & G_C^\ell & 0 & G_C^h \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{y}_\ell \\ \mathbf{y}_h \\ \mathbf{n}_h \end{pmatrix} + \begin{pmatrix} \mathbf{e}_+ \\ \mathbf{e}_- \\ \mathbf{e}_C \end{pmatrix}$$

$\mathbf{d}_A, \mathbf{d}_B, \mathbf{d}_C$ are data of Swarm A, B, C

$\mathbf{d}_+ = \frac{\mathbf{d}_A + \mathbf{d}_B}{\sqrt{2}}, \mathbf{d}_- = \frac{\mathbf{d}_A - \mathbf{d}_B}{\sqrt{2}}$ are **sum** and **difference** of Swarm A,B (lower pair)

$G_+, G_-, G_C, \mathbf{x}$ are design matrices and vector of all model parameters but crustal field, these are sensed by all data

$G_+^\ell, G_-^\ell, G_C^\ell, \mathbf{y}_\ell$ are design matrices and vector of low-order crustal field model parameters, sensed by all data

G_-^h, \mathbf{y}_h are design matrix and model vector of high-order crustal field, sensed by \mathbf{d}_-

$G_+^h, G_C^h, \mathbf{n}_h$ are design matrices and model vector of high-order *crustal-nuisance* field ("garbage"), sensed by \mathbf{d}_+ and \mathbf{d}_C

LITHOSPHERIC FIELD RECOVERY

The recovery of the lithospheric field is assessed by comparing the estimated coefficients of the crust with the coefficients of the input model. Fig. 3 shows the degree error evaluated at ground level and the degree correlation for three different models:

- "Field only" which does not take explicit advantage of the gradient information (no SIVW)
- "Field + Gradient, Baseline" which makes use of the gradient information but only uses data above 300 km altitude (baseline mission)
- "Field + Gradient, Extended" which also uses two months of data from Swarm A and B with altitudes between 250 and 300 km altitude (extended mission)

Fig. 3: Lithospheric Degree Error and Correlation

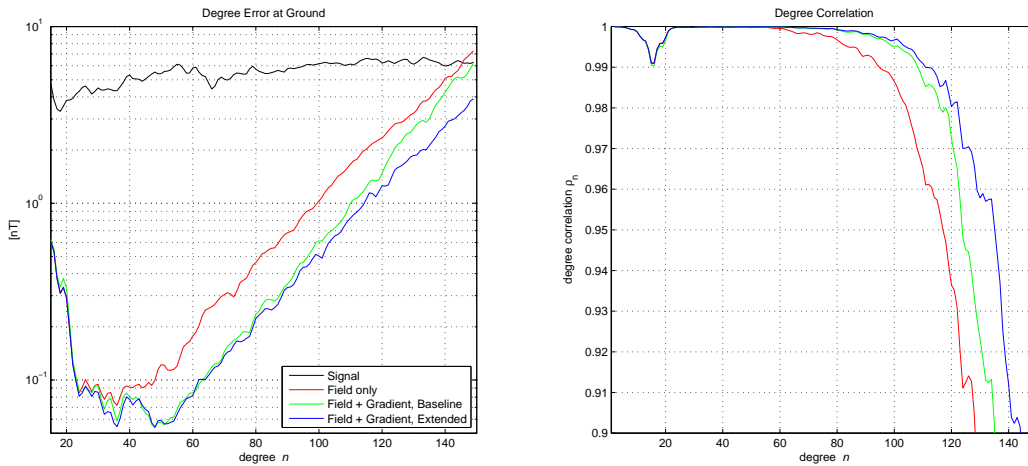


Fig. 4: Sensitivity Matrix for Lithospheric Recovery

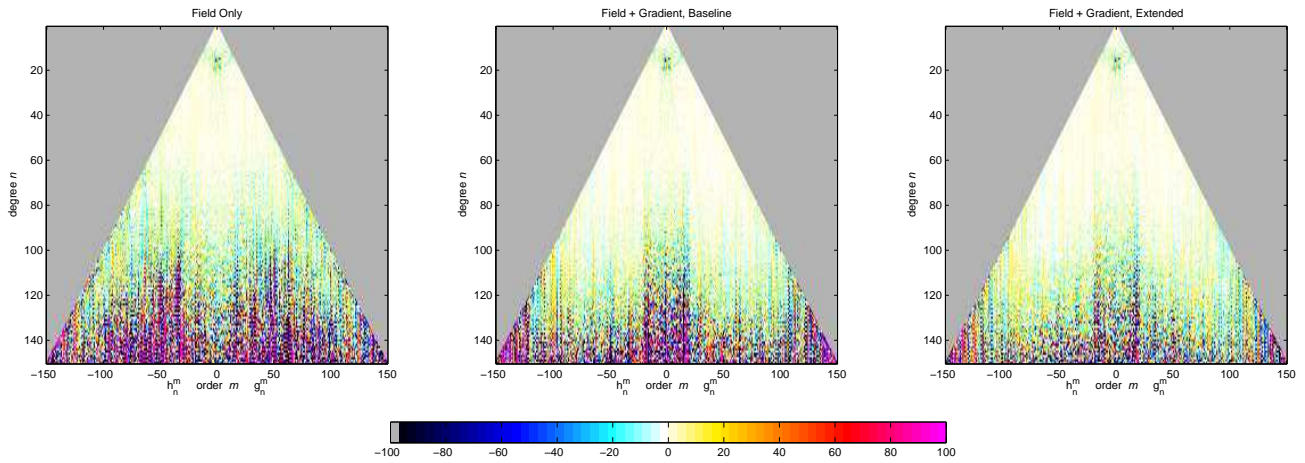
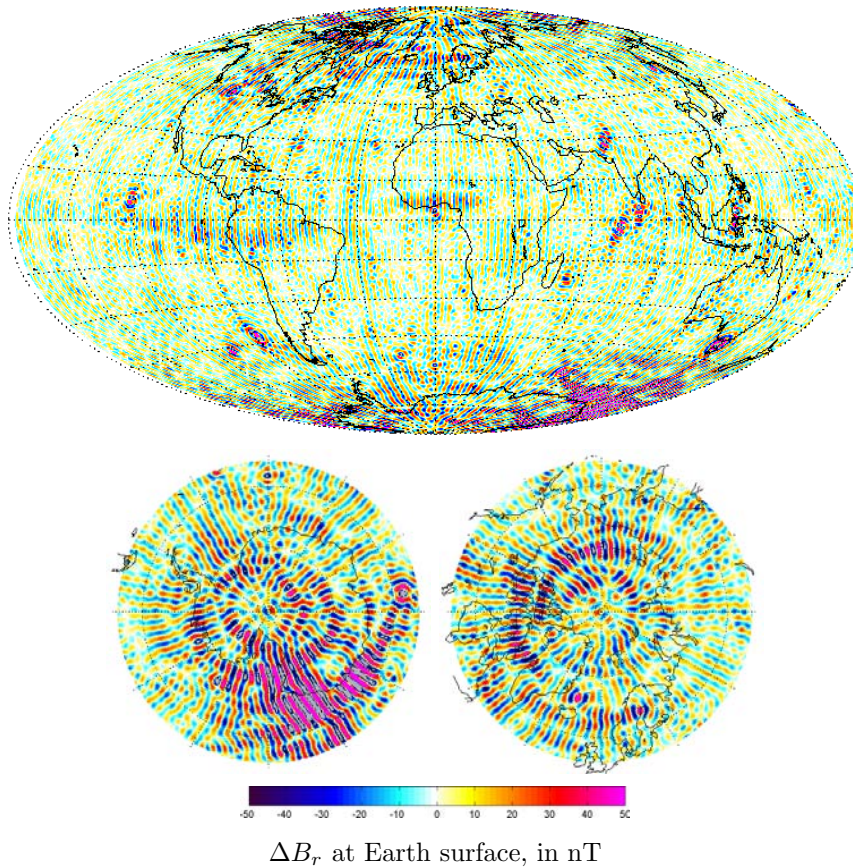


Fig. 4 shows the sensitivity matrix – the parameter recovery for every degree and order – for the three models. From Fig. 3 and 4 the improvements of the gradient exploitation as well as the extended mission data are obvious, the former for degrees above 40 and the latter for degrees above 100-120.

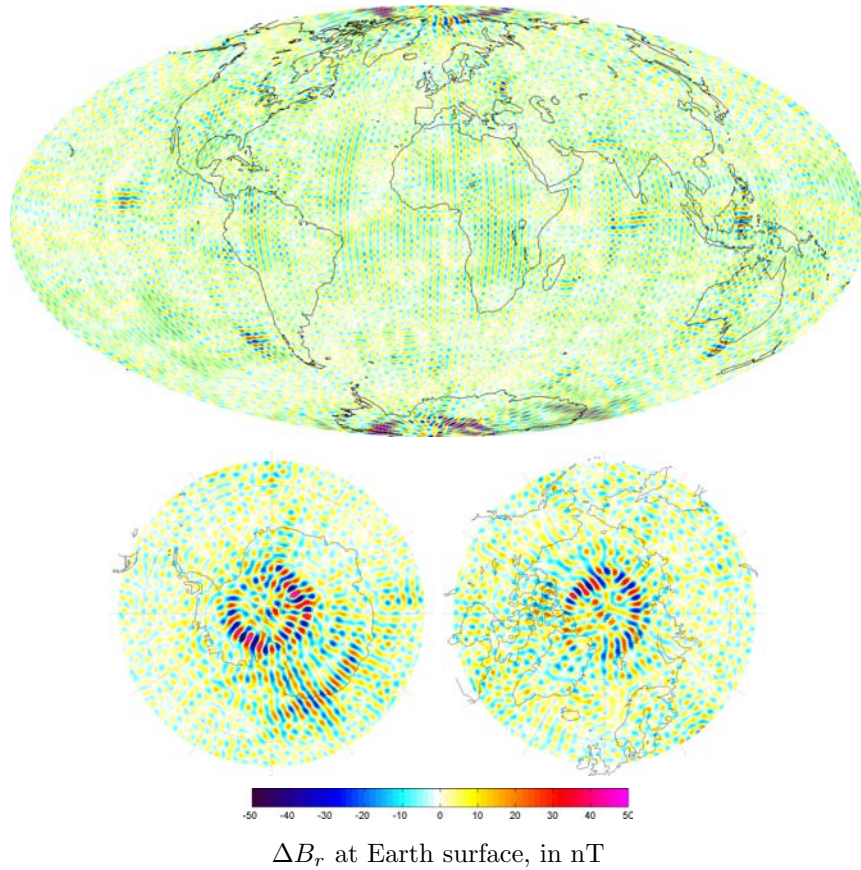
Global maps of the lithospheric field differences (recovered – input model) for the “Field + Gradient, Baseline” and “Extended” models are shown in Fig. 5 and Fig. 6. From these maps, the gain of the extended mission is clear.

Fig. 5: Maps of Lithospheric Field Differences, Model: Field + Gradient, Baseline



ΔB_r at Earth surface, in nT

Fig. 6: Maps of Lithospheric Field Differences, Model: Field + Gradient, Extended



REGULARIZATION OF SECULAR VARIATION (SV)

When investigating the power spectrum of the Secular Variation (SV) recovered using an unconstrained model it is much too energetic compared e.g. to CHAOS-2 [2], especially for high degrees, see green curve ($\lambda = 0$) on the left plot of Fig. 7.

Fig. 7: SV Power as Function of Damping Weight

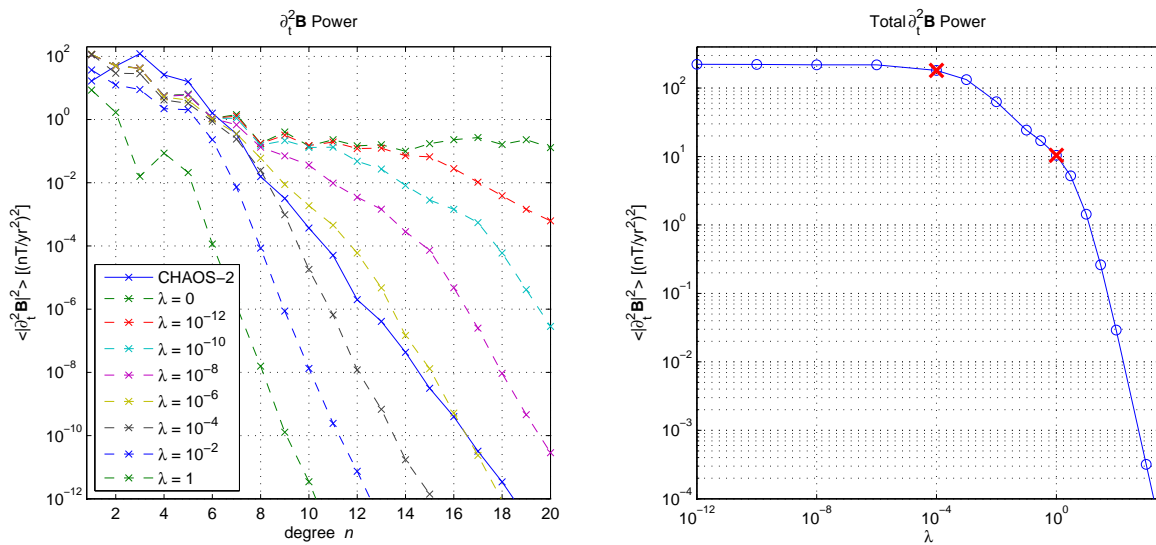
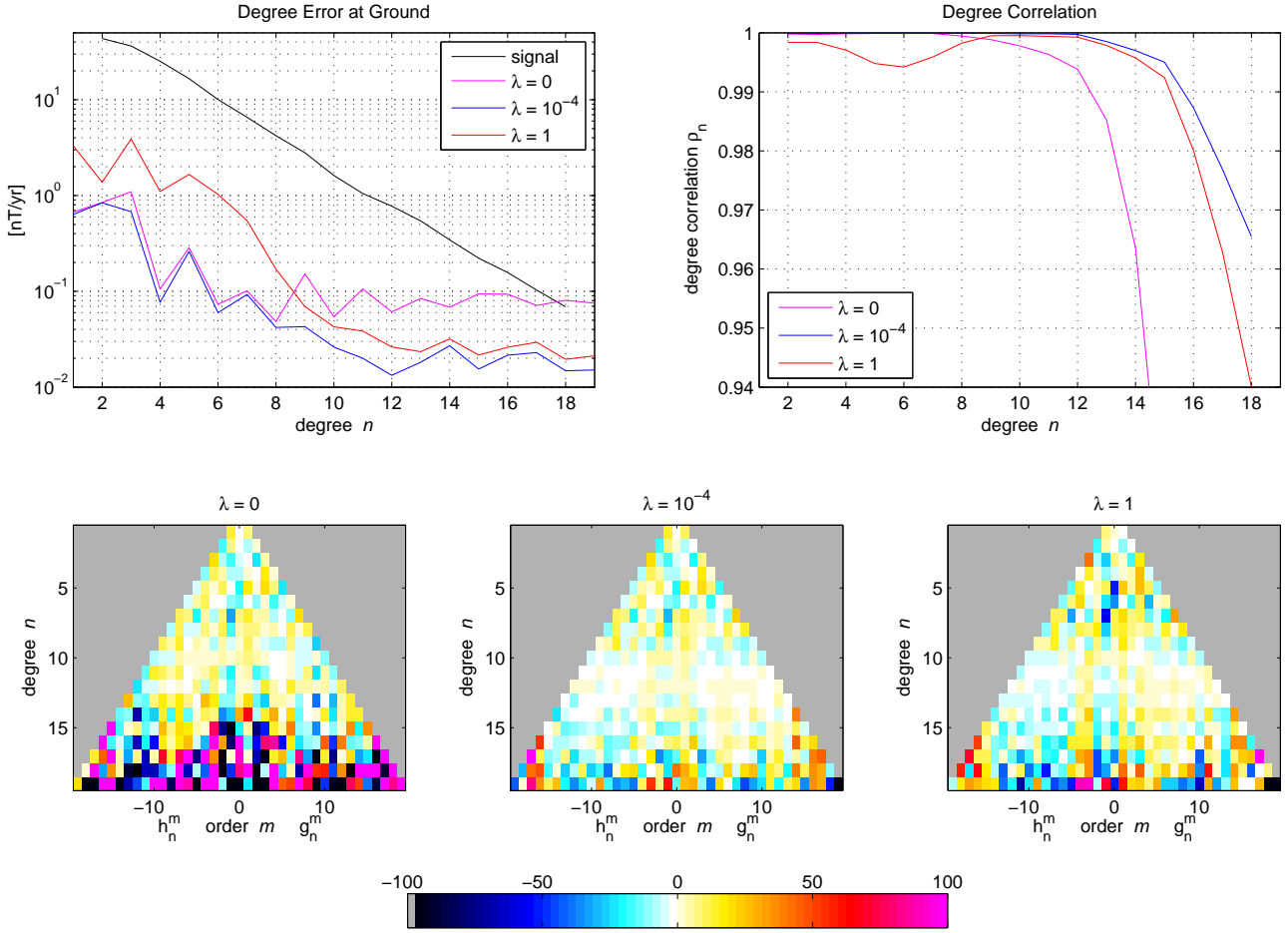


Fig. 8: SV Recovery for Different Regularizations



This effect is damped by introducing a regularization of \ddot{B}_r at the core-mantle boundary. The effect of this regularization on the SW power, the power of $\partial_t^2 \mathbf{B}$ at Earth surface, for various weights (λ) are plotted in Fig. 7 – per degree (left) and total for all degrees up to 20 (right). Two specific models marked by red crosses in Fig. 7 right are analyzed further. In the top part of Fig. 8 the degree error and degree correlation of the SV are shown for three models: $\lambda = 0$ (unconstrained model), $\lambda = 10^{-4}$, and $\lambda = 1$. The unconstrained model have clear deficiencies above degree 8 whereas the highly constrained model ($\lambda = 1$) have some trouble especially for low degrees. This is confirmed by the sensitivity matrices shown in the bottom of Fig. 8.

Consequently the optimal choice of the regularization weight is around $\lambda = 10^{-4}$, i.e. the factor for which the SV power spectrum is close to that of CHAOS-2, see Fig. 7.

REFERENCES

- [1] N. Olsen, et al, “Swarm - End-to-End mission performance simulator study”, ESA contract No 17263/03/NL/CB. *Report 1/2004*, Danish Space Research Institute, Copenhagen, 2004.
- [2] N. Olsen, M. Manda, T.J. Sabaka, L. Tøffner-Clausen, “CHAOS-2 – A Geomagnetic Field Model Derived from one Decade of Continuous Satellite Data”, unpublished