
Methodology and Analytical Basis for the Assessment of Cracked Components Using the Program VERB

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Workshop on Fracture Control of Spacecraft, Launchers and
their Payloads and Experiments

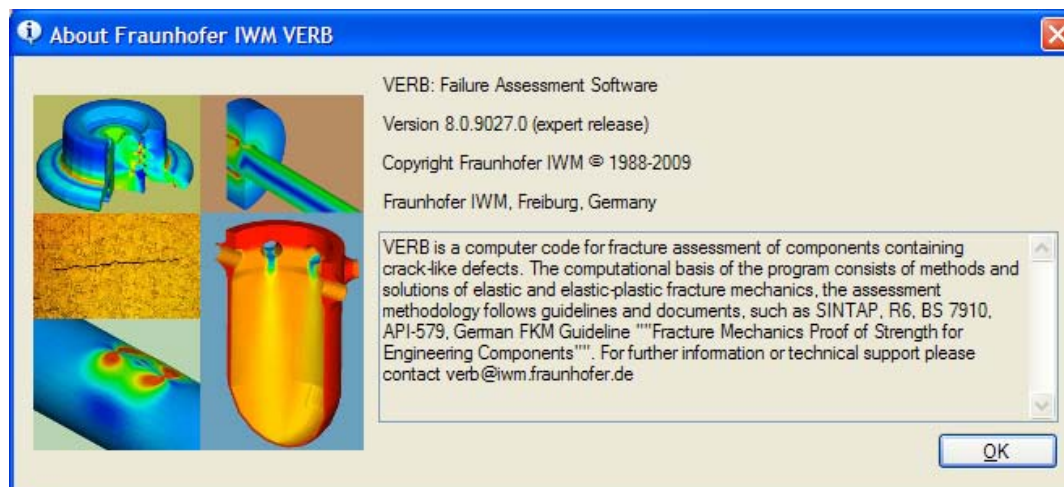
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Overview

- Introduction to IWM VERB
- Scope and application areas
- Assessment methodology
- Background of analytical tools
- Prospective developments
- Application examples

Introduction

- VERB = „Versagensbewertung von rissbehafteten Bauteilen“ standing for “Failure Assessment of Cracked Components”
- Developed at IWM since 1988 as IWM Beta → ... → IWM VERB
- Use both for internal purposes (research, industrial projects) and by external customers
- Comprises state-of-the-art experience of fracture mechanics codes and own research results
- Since 11/2007 Version 8.0, last release 12/2008

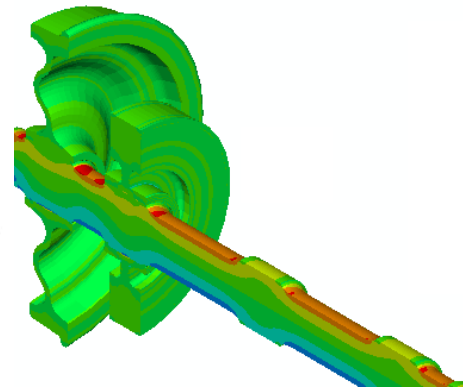
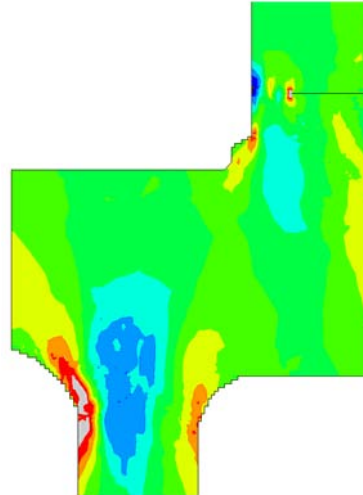
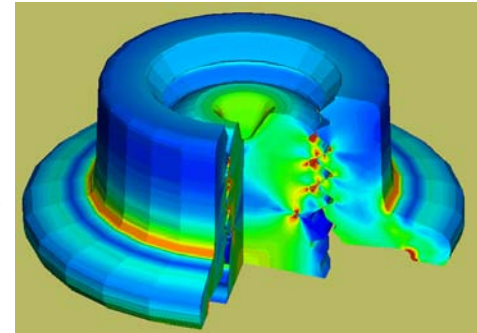
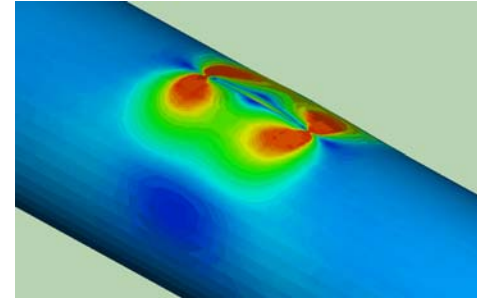
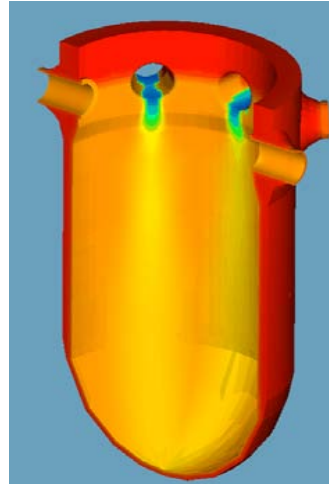


Scope

- Fracture mechanics assessment of detected or postulated crack-like defects in components subjected to static or cyclic loading
- Employs elastic stresses derived analytically (simple components and load cases) or numerically for the crack-free component
- Assessment at static loading using the FAD approach, both crack initiation and crack instability analyses, determination of critical conditions and safety margins
- Leak-before-break (LBB) and leak area calculations for pressurized components
- Fatigue crack growth analysis at constant and variable amplitude loading (no load interaction models are currently implemented)
- Semi-probabilistic calculations on the basis of partial safety factor methodology (BS 7910 or API 579)

Application areas

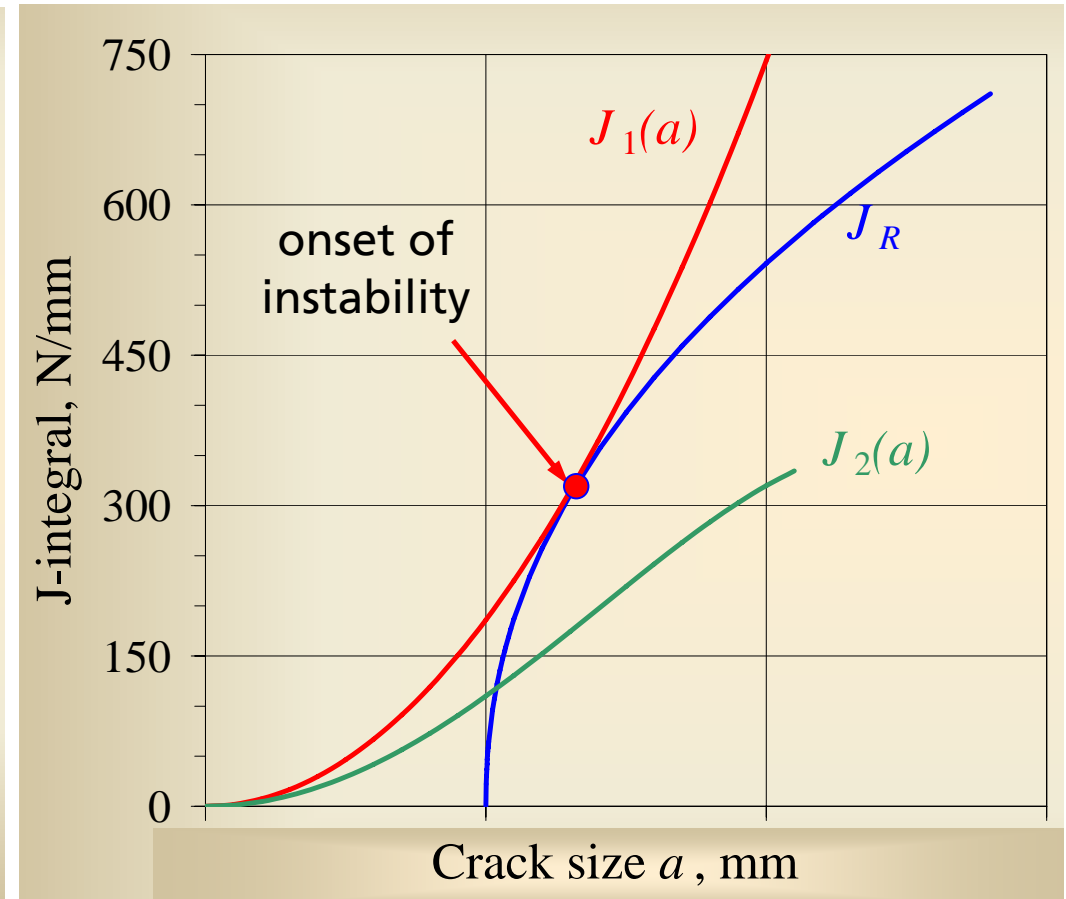
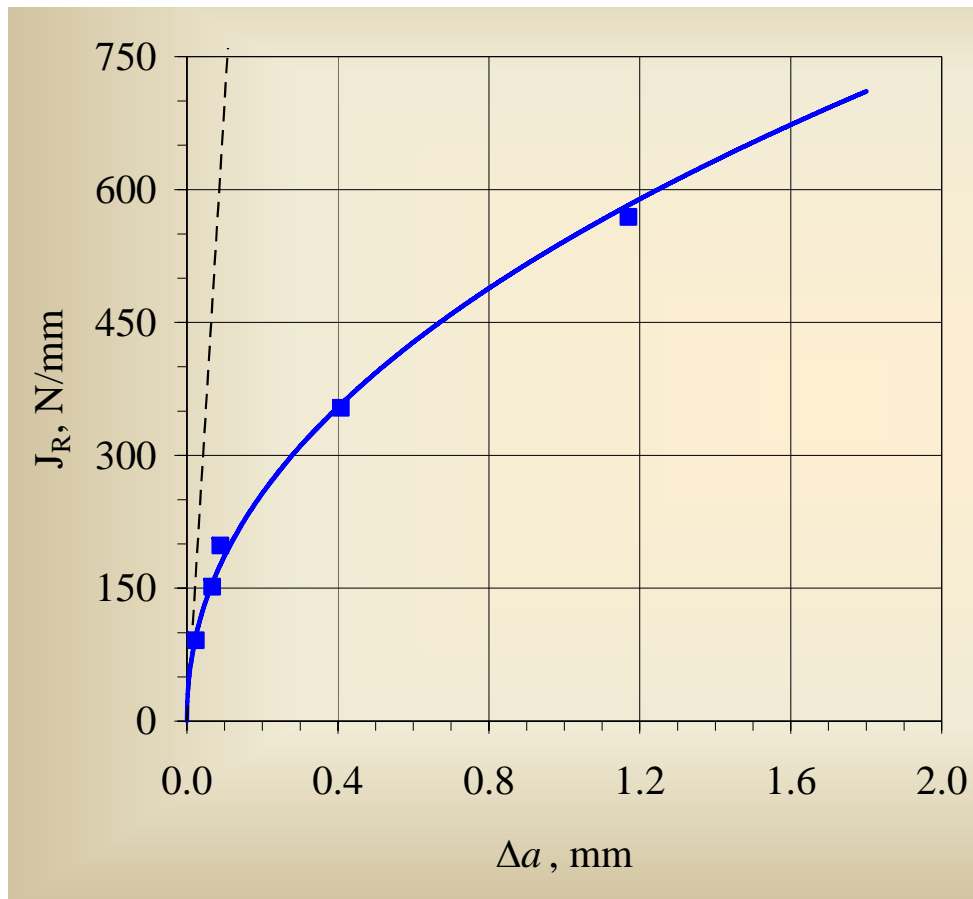
- Power generation
- Aerospace
- Pipeline
- Chemical production
- Railway
- Technical expertise
- Insurances
- Education



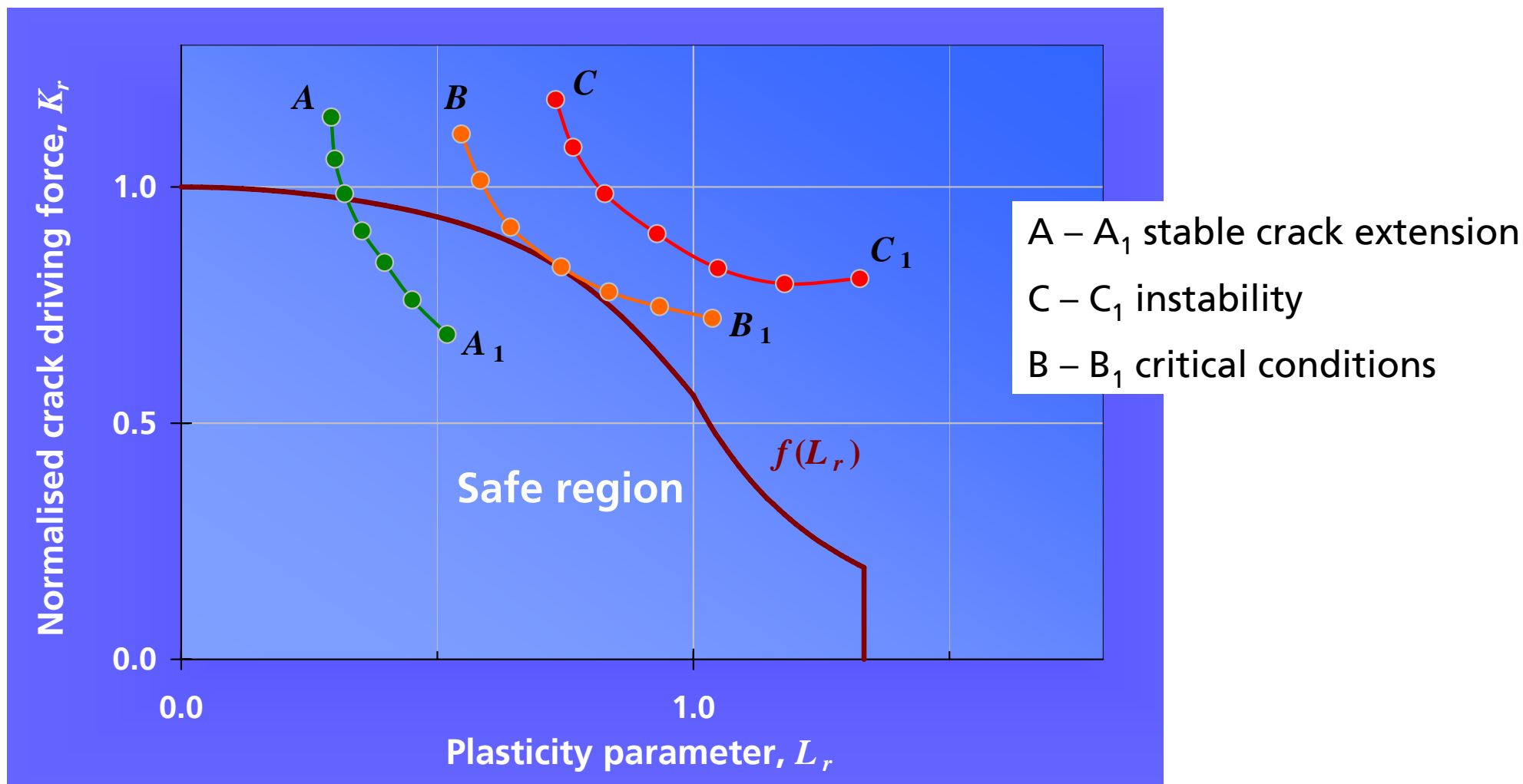
Reference documents

- SINTAP
- R6
- BS 7910
- German FKM-Guideline “Fracture Mechanics Proof of Strength”
- API 579
- FITNET
- Papers in periodicals
- Own research

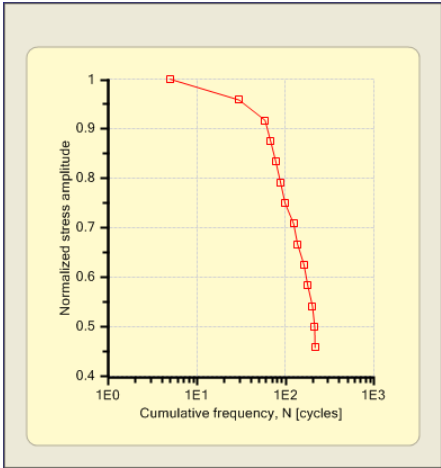
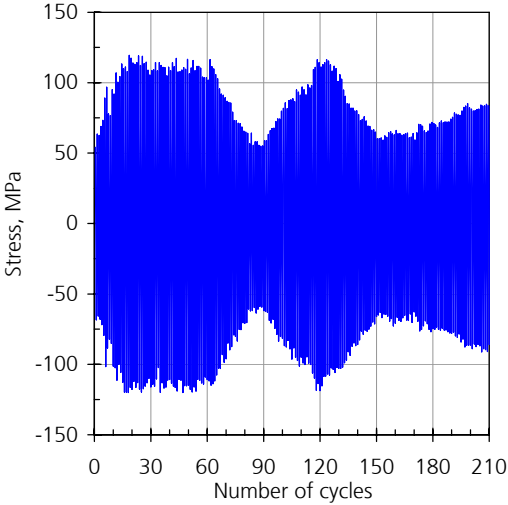
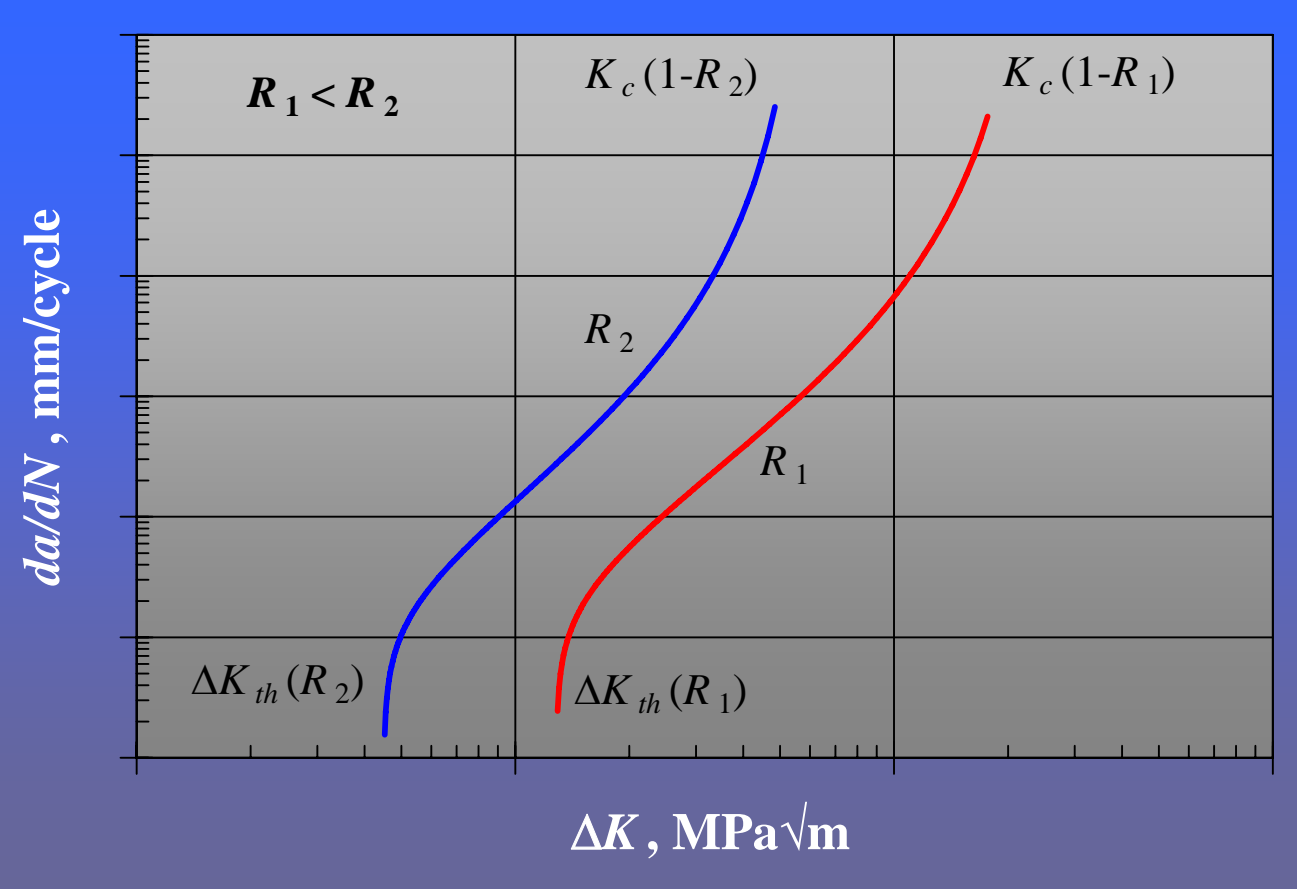
Stable crack growth, J-R curve



FAD assessment at static loading – crack instability



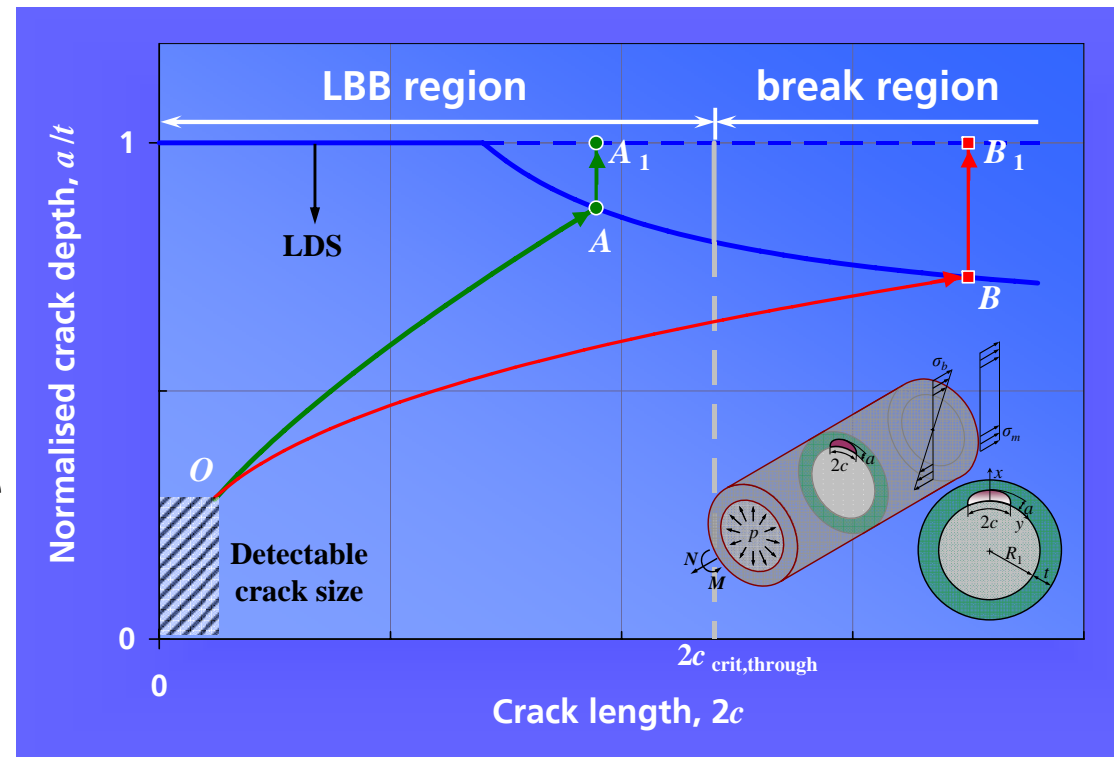
Fatigue crack growth calculations



Leak-before-break analysis (schematic)

Basic computational steps:

1. Calculate critical length of a through-thickness flaw, $2c_{\text{crit,through}}$
2. Calculate critical length of a surface flaw, $2c_{\text{crit}}$, at varying crack depth, $0 < a < t$
3. Use 1 and 2 to build the limit curve
4. Assess crack growth for a postulated or detected flaw under in-service conditions
5. Evaluate LBB behavior:
A = leak, B = break



Partial safety factors (PSF)

- FAD assessment using scattered and/or uncertain data
- Input data described by the mean and coefficient of variation
- Usage through modification the mean values and subsequent deterministic calculations
- Application to crack initiation assessment
- Application to crack instability is possible but still not verified
- Derivation based on probabilistic calculations assuming specific types of distribution functions
- Included in failure assessment procedures BS 7910 (same approach in SINTAP, German FKM Guideline, FITNET) and API 579
- Different approaches → different results → expert knowledge is required

Definition of PSF in BS 7910

Parameter, PSF	V	P_f				
		0.23	10^{-3}	7×10^{-5}	10^{-5}	10^{-7}
Crack size, γ_a	0.1	1.0	1.4	1.5	1.7	2.1
	0.2	1.05	1.45	1.55	1.8	2.2
	0.3	1.08	1.5	1.65	1.9	2.3
	0.5	1.15	1.7	1.85	2.1	2.5
Applied stress, γ_σ	0.1	1.05	1.2	1.25	1.3	1.4
	0.2	1.1	1.25	1.35	1.4	1.55
	0.3	1.12	1.4	1.5	1.6	1.8
Yield strength, γ_{Re}	0.1	1.0	1.05	1.1	1.2	1.5
Fracture toughness, γ_{Kmat}	0.1	1.0	1.3	1.5	1.7	2.0
	0.2	1.0	1.8	2.6	3.2	5.5
	0.3	1.0	2.85	2.6 ⁽¹⁾	3.2 ⁽¹⁾	5.5 ⁽¹⁾

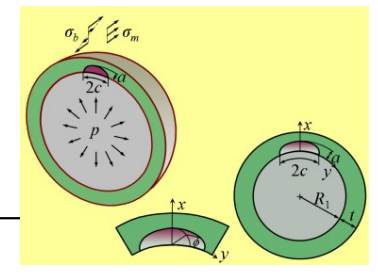
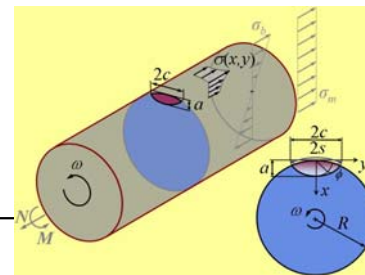
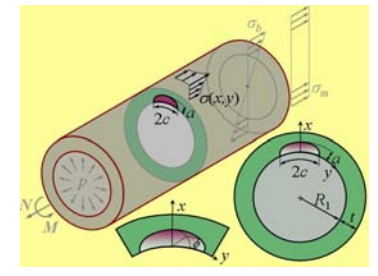
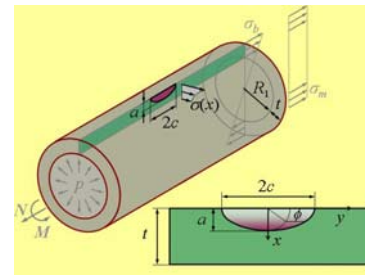
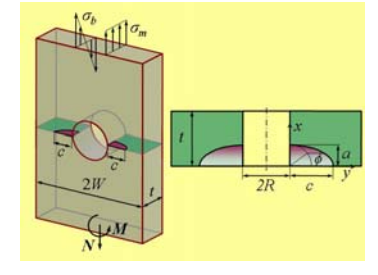
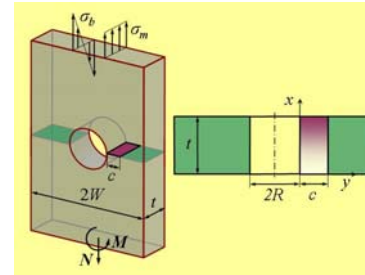
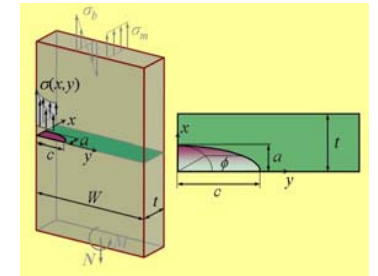
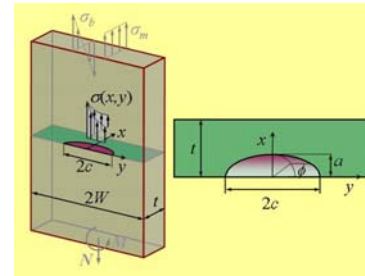
$$a = \gamma_a \mu_a, \quad \sigma_p = \gamma_\sigma \mu_{\sigma_p}, \quad R_e = \frac{1}{\gamma_{Re}} (\mu_{Re} - 2\sigma_{Re}), \quad K_{mat} = \frac{1}{\gamma_{Kmat}} (\mu_{Kmat} - \sigma_{Kmat})$$

Analytical solutions in VERB

- FAD assessment line according to SINTAP (different options)
- Stress intensity factors, K_I
 - polynomial influence functions
 - weight functions
 - closed form equations – only for a few models
- Plastic limit load parameter L_r
 - closed form analytical equations
 - analytical equations to be solved iteratively
- Where appropriate, possibility to vary K_I and L_r solutions to achieve an optimal analysis performance (different accuracy and validity ranges for different solutions, comparison of different codes)

Analytical models in VERB

Component	Crack	Location	Orientation	Load components	Stress profile 1D	Stress profile 2D ⁽²⁾
plate	extended surface	-	-	•	•	-
	extended embedded	-	-	•	•	-
	semi-elliptical surface	-	-	•	•	⊕
	semi-elliptical embedded ⁽³⁾	-	-	•	•	⊕
	quarter-elliptical corner	-	-	•	•	⊕
	elliptical embedded	-	-	•	•	⊕ ⁽¹⁾
through-thickness	-	-	•	-	-	
plate with hole	semi-elliptical, one crack	bore surface	-	•	-	-
	semi-elliptical, two cracks	bore surface	-	•	-	-
	quarter-elliptical, one crack	bore surface	-	•	-	-
	quarter-elliptical, two crack	bore surface	-	•	-	-
	through-thickness, one crack	bore surface	-	•	-	-
through-thickness, two cracks	bore surface	-	•	-	-	
hollow cylinder	extended surface	internal	circumferential	•	•	-
		external	circumferential	•	•	-
	extended embedded	internal	axial	•	•	-
		external	axial	•	•	-
	semi-elliptical surface	internal	circumferential	•	•	⊕
		external	circumferential	•	•	⊕
	elliptical embedded	internal	axial	•	•	⊕
		external	axial	•	•	⊕
	through-thickness	-	circumferential	•	•	⊕ ⁽¹⁾
		-	axial	•	•	⊕ ⁽¹⁾
round bar	extended surface	-	circumferential	•	-	-
	circular embedded	-	circumferential	•	-	-
	semi-elliptical surface	-	circumferential	•	•	⊕
rotating round bar	semi-elliptical surface	-	circumferential	•	•	⊕
sphere	extended surface	internal	-	•	•	-
	extended embedded	external	-	•	•	-
	semi-elliptical surface	internal	-	•	•	-
		external	-	•	•	-
elliptical embedded	-	-	•	•	⊕ ⁽¹⁾	
through-thickness	-	-	•	•	-	

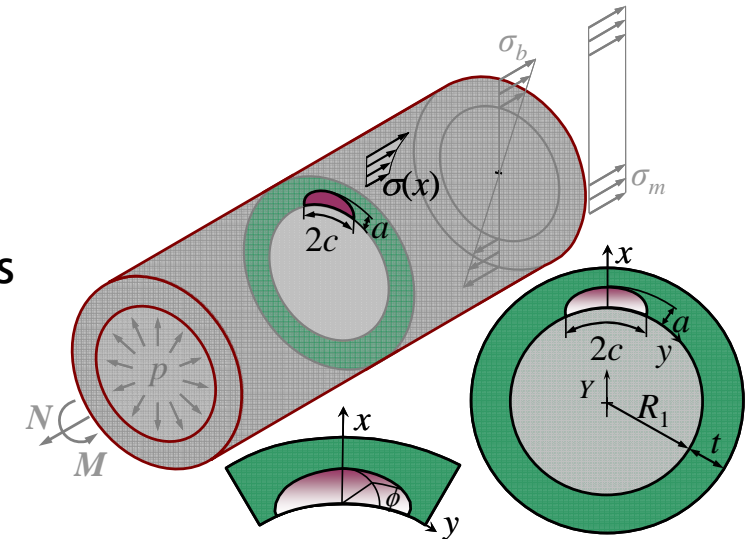


K_I solutions for surface cracks in cylinders

Solutions in SINTAP (1999), CEA (2000), DNV-Code:

$$\sigma(x) = \sum_{m=0}^3 D_m \left(\frac{x}{t} \right)^m \quad \text{for axial cracks}$$

$$\sigma(x) = \sum_{m=0}^3 D_m \left(\frac{x}{t} \right)^m + \sigma_b \frac{Y}{R_2} \quad \text{for circumferential cracks}$$



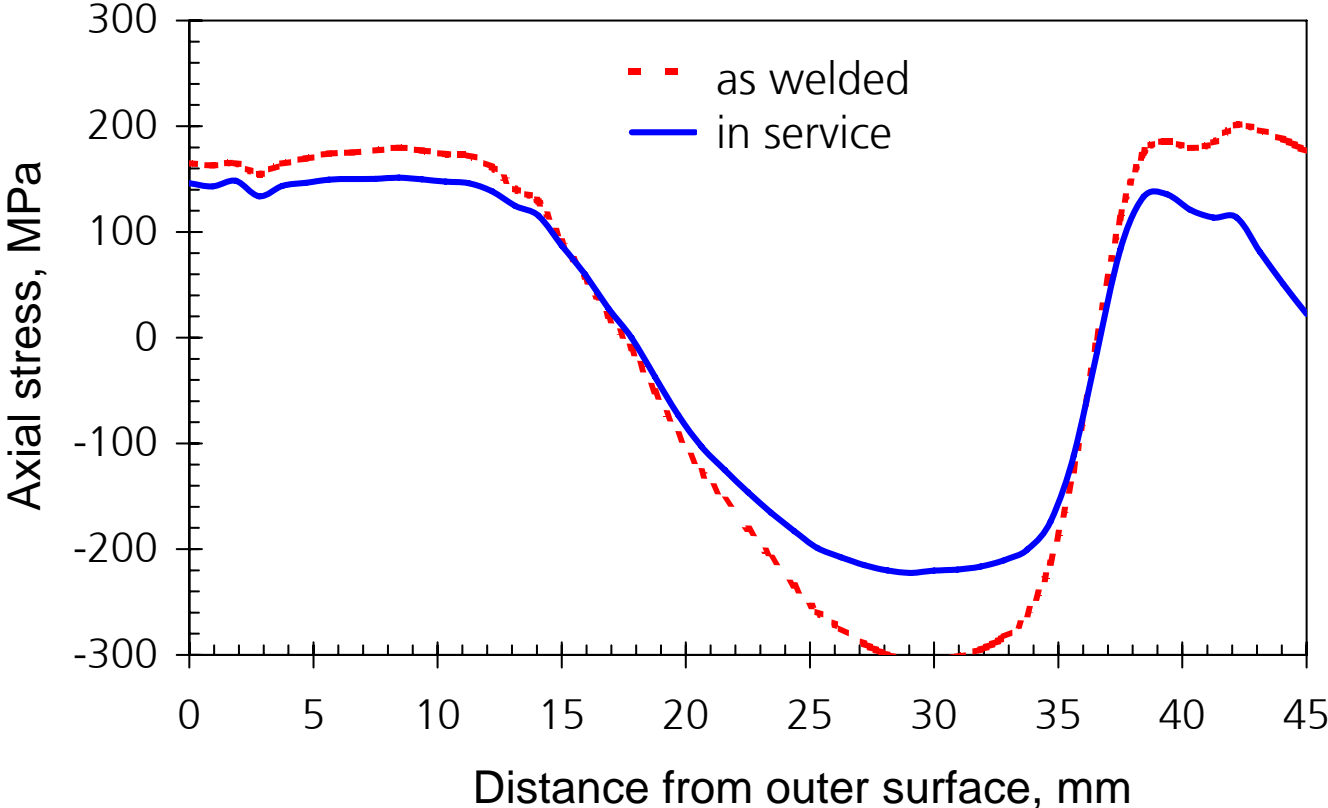
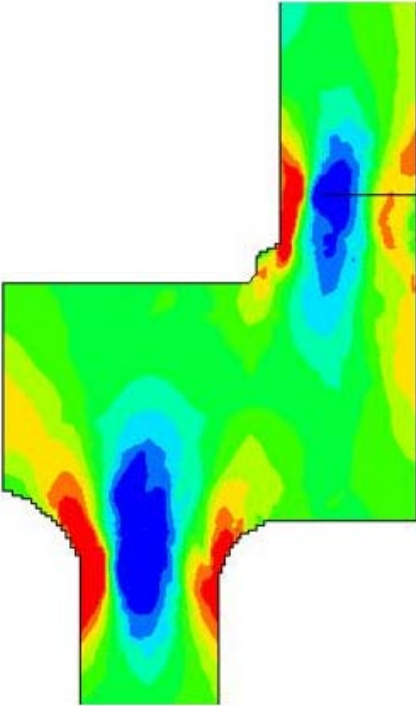
Stress intensity factor:

$$K \left(\frac{a}{c}, \frac{a}{t}, \frac{t}{R_1}, \phi \right) = \left[\sum_{m=0}^3 D_m f_m \left(\frac{a}{c}, \frac{a}{t}, \frac{t}{R_1}, \phi \right) \left(\frac{a}{t} \right)^m + \sigma_b F_b \right] \sqrt{\pi a}$$

Influence functions f_m are calculated by FEM and tabulated for $\phi = 0^\circ$ and 90° .

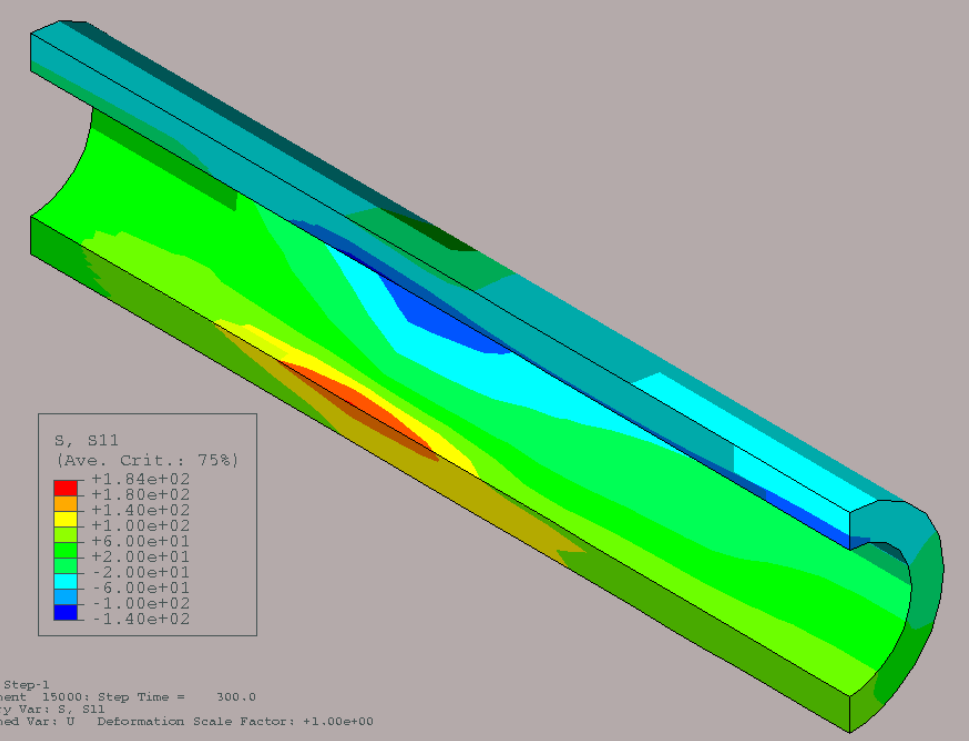
Examples of non-uniform stress profiles (1D)

Circumferential weld in a cylindrical shell
Axially symmetric stress distribution (more than 2 extrema)

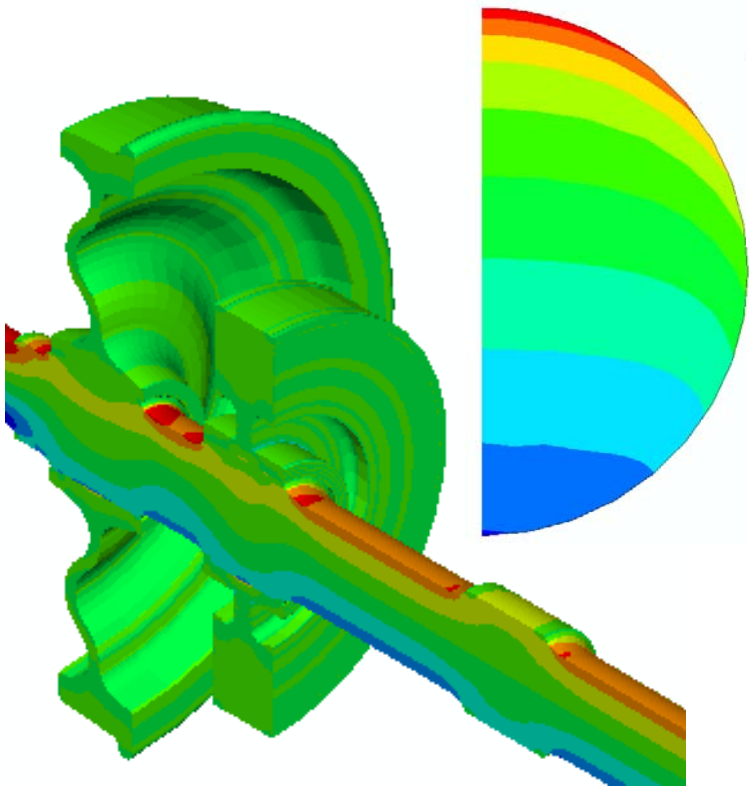


Examples of non-uniform stress profiles (2D)

Thermal mixing in piping
2D axial stress distribution in a prospective crack plane



Rotating round bar
2D, non-symmetric stress field in crack plane



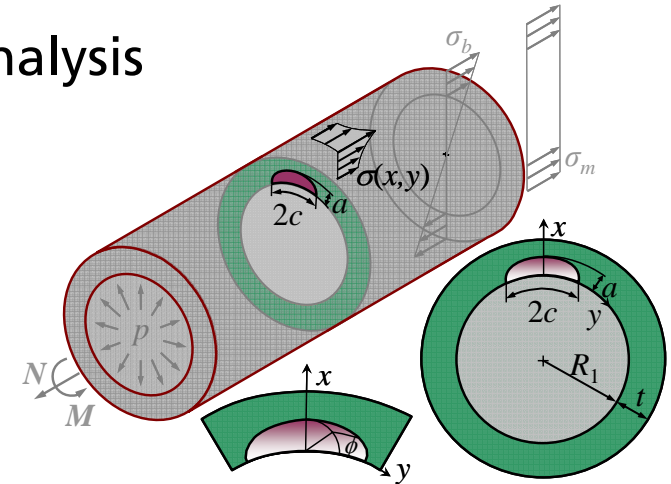
Polynomial influence functions, 2D stress field

Solution by IWM (1995, 2000), boundary-element analysis

$$\sigma(x, y) = \sum_{m=0}^4 \sum_{n=0}^4 \left[D_{mn}^{(1)} \left(\frac{x}{t} \right)^m \left| \frac{y}{t} \right|^n + D_{mn}^{(2)} \left(\frac{x}{t} \right)^m \operatorname{sgn}(y) \left| \frac{y}{t} \right|^n \right]$$

Stress intensity factor:

$$K \left(\frac{a}{c}, \frac{a}{t}, \frac{t}{R_1}, \phi \right) = \left[\sum_{m=0}^4 \sum_{n=0}^4 \left(D_{mn}^{(1)} f_{mn}^{(1)} + D_{mn}^{(2)} f_{mn}^{(2)} \right) \left(\frac{a}{t} \right)^{m+n} \left(\frac{a}{c} \right)^{-n} \right] \sqrt{\pi a}$$

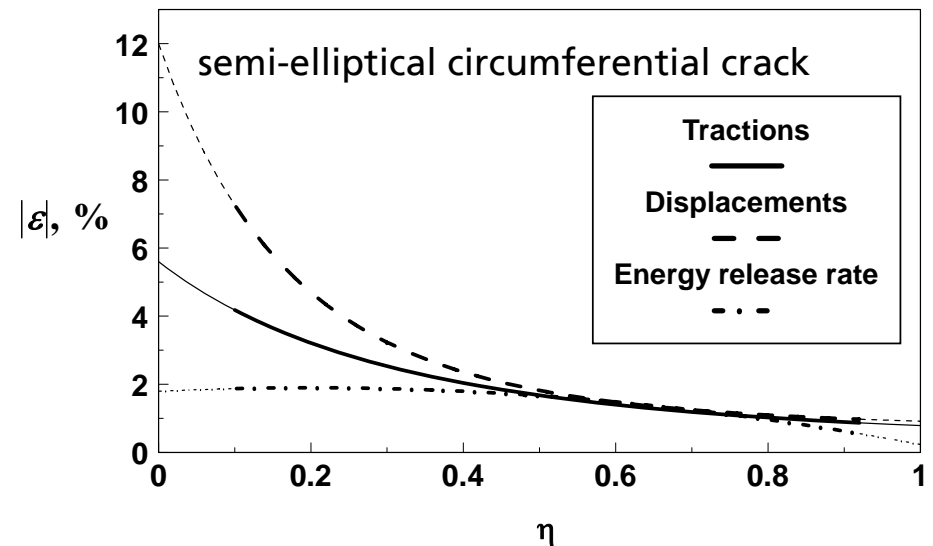


Features

- Flexible stress fit in the crack plane
- Verified solution accuracy through evaluation of energy integrals

$\eta = 0$ at the surface point

$\eta = 1$ at the crack apex



Limit load solutions

References: SINTAP, API 579, DNV, "Ductile Fracture Handbook", periodicals

Examples for semi-elliptical cracks in a cylinder:

- axial cracks
$$L_r = \frac{\sigma_{mL}}{(1 - \xi^{3.11})^{0.95} R_e}, \quad \xi = \frac{ac}{t(t+c)}$$

- circumferential cracks

$$L_r = \sqrt{\frac{\left(\frac{\sigma_{mL}}{S'_m}\right)^2 + \left(\frac{\sigma_{bL}}{S'_b}\right)^2}{\left(\frac{S_m}{S'_m}\right)^2 + \left(\frac{S_b}{S'_b}\right)^2}}$$

$$\frac{S_m}{R_e} = 1 - 2\frac{\beta}{\pi} - \frac{a}{t}\frac{\alpha}{\pi}, \quad \frac{S_b}{R_e} = \frac{4}{\pi}\sin\beta - \frac{2}{\pi}\frac{a}{t}\sin\alpha$$

$$\begin{aligned} \sigma_{mL}S_b - \sigma_{bL}S_m &= 0 \\ S'_m = S_m &\text{ for } S_b = 0 \\ S'_b = S_b &\text{ for } S_m = 0 \end{aligned}$$

$$\alpha = \begin{cases} \frac{c}{R_1} & \text{for } \frac{c}{R_1} \leq \pi - \beta \\ \pi - \beta & \text{for } \frac{c}{R_1} > \pi - \beta \end{cases}$$

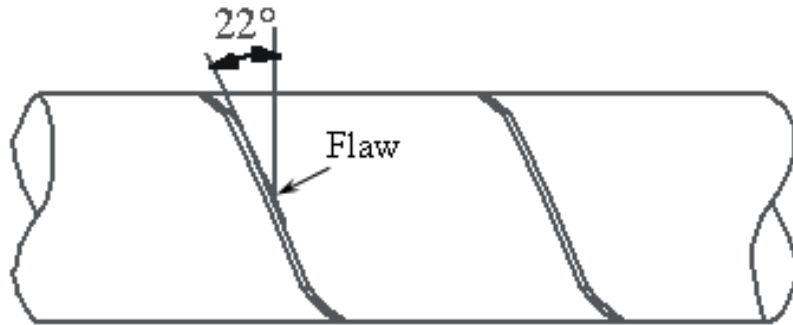
General note: most of limit load solutions are still very conservative
 → large potential for improving the assessment accuracy

Open issue: multi-axial state of stress

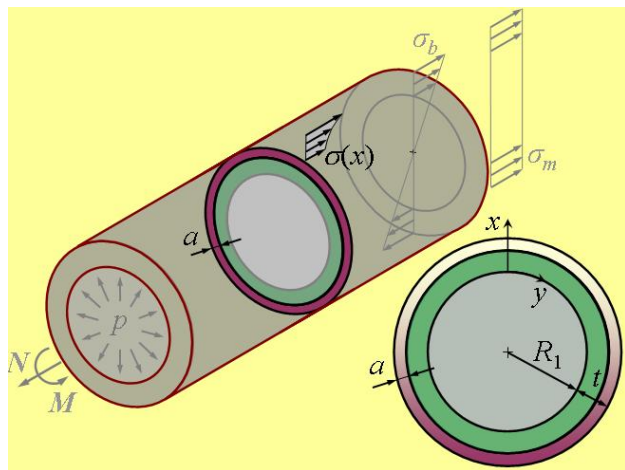
Overview of extension topics

- Extended analytical models – 2D stress fields in the cracked section
- Transient analyses – time dependent stress fields, temperature dependent material properties
- Probabilistic calculations
- Code specific safety factors and assessment lines
- Consideration of the crack tip constraint in FAD assessment

Fitness-for-service assessment of spiral welded pipe



Model:

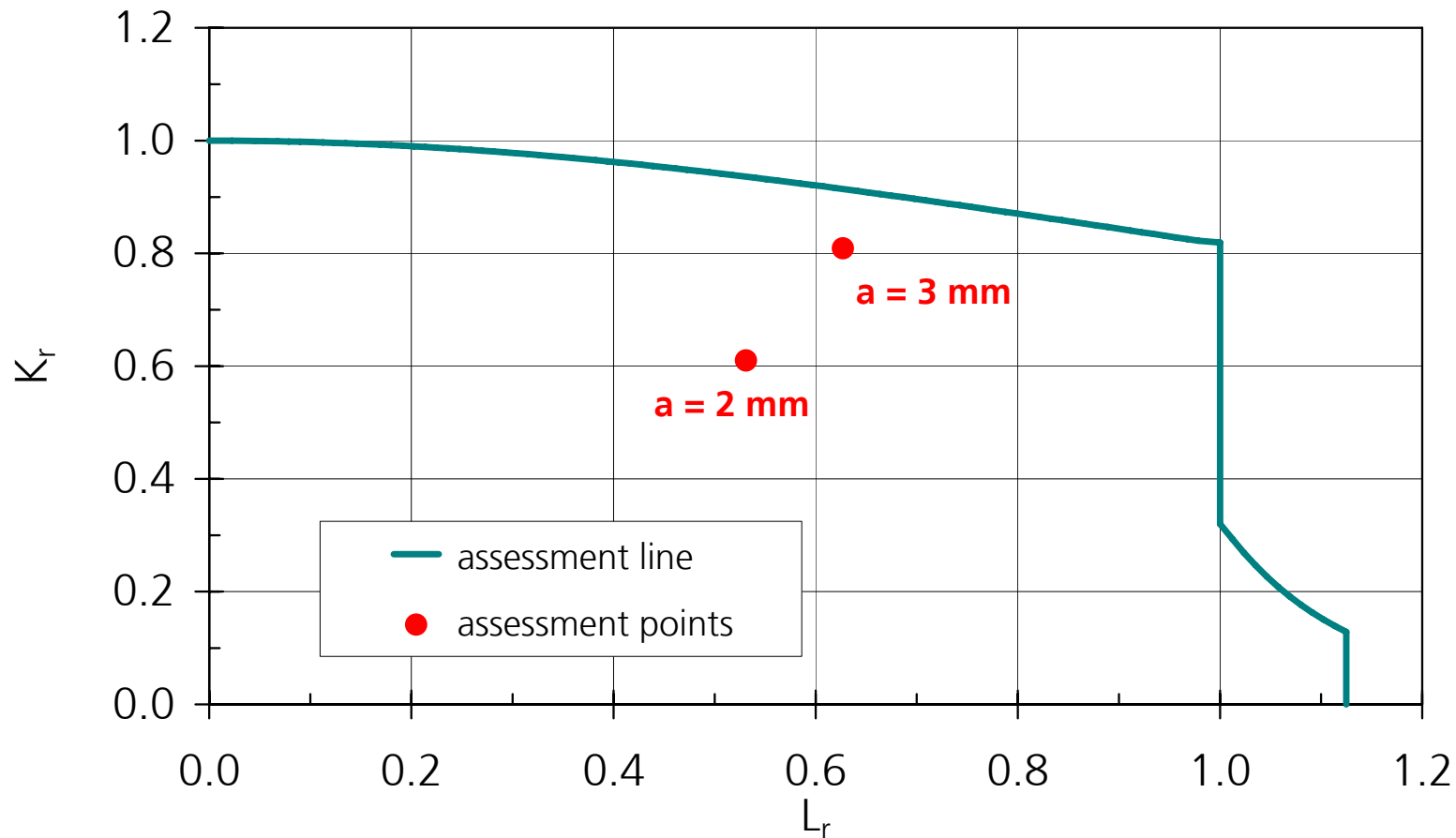


Outer diameter, D_a	1420 mm
Wall thickness, t	15.6 mm
Crack depth, a	2 to 3 mm
Yield strength, $R_{p0.2}$	480 MPa
Tensile strength, R_m	600 MPa
Fracture toughness, K_{mat}	91 MPa \sqrt{m} ⁽¹⁾
Internal pressure, p	7.6 MPa
Primary stress, σ_p	192 MPa ⁽²⁾
Secondary stress, σ_s	288 MPa ⁽³⁾

- 1) estimated as the lowest value from 10 CTOD tests
- 2) membrane stress acting normal to the weld line
- 3) conservativ estimate as 60% of the yield strength

Spiral welded pipe, deterministic assessment

Deterministic result: component is safe



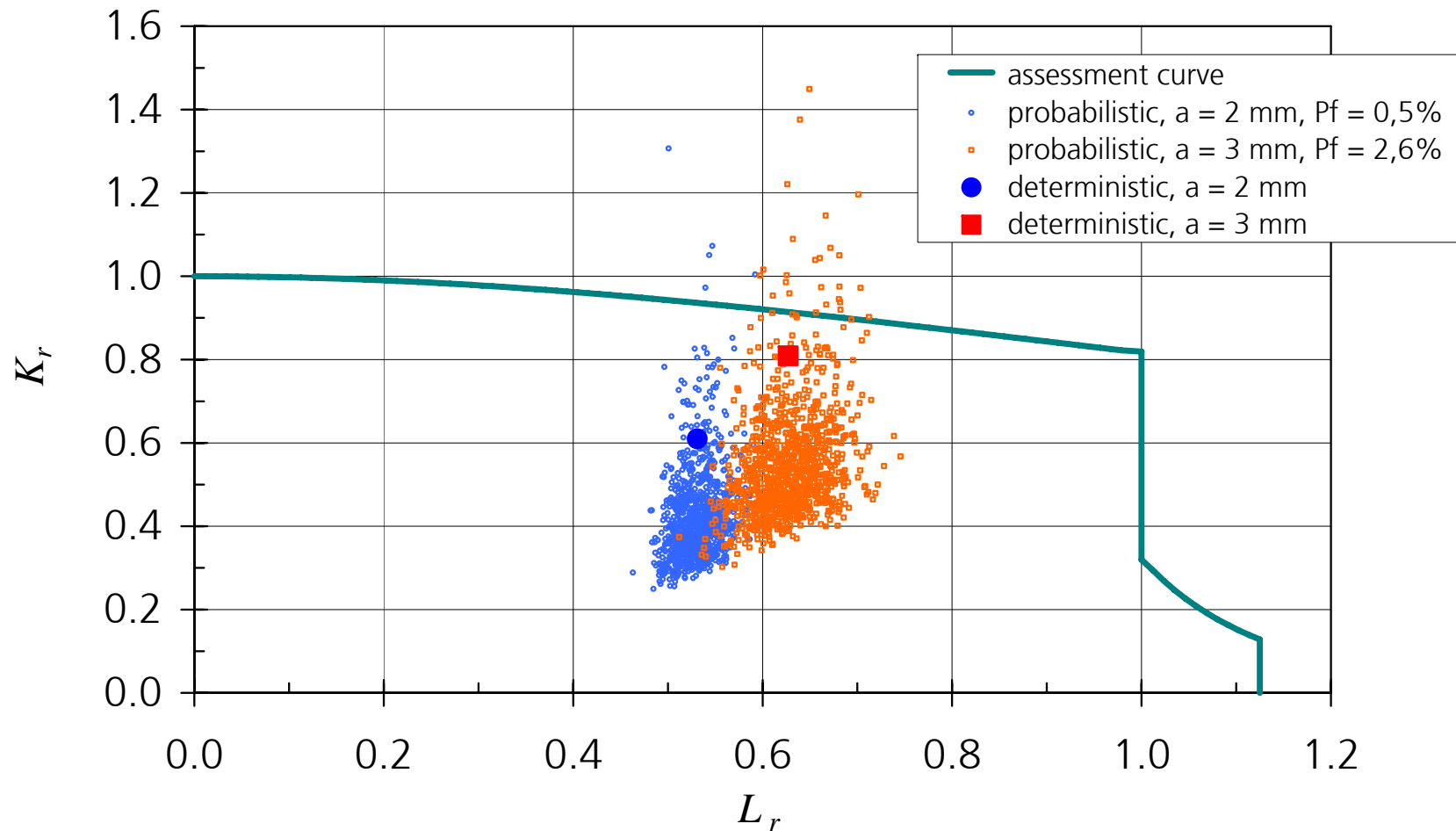
Spiral welded pipe, scattered data

Input data	Mean value	Distribution type	Distribution parameters
Crack depth a , mm	2 3	normal	$\mu = 2, \sigma = 0,2$ $\mu = 3, \sigma = 0,3$
Primary stress σ_p , MPa	192	normal	$\mu = 192, \sigma = 4$
Secondary stress σ_s , MPa	288	lognormal	$x_0 = 282, m = 6, \sigma = 0,6$
Yield strength $R_{p0.2}$, MPa	480	lognormal	$x_0 = 474, m = 6, \sigma = 0,4$
Tensile strength R_m , MPa	600	normal	$\mu = 600, \sigma = 10$
Fracture toughness K_{mat} , MPa \sqrt{m}	91	Weibull	$x_0 = 20, \beta = 4, \eta = 149$

$$F(x) = 1 - \exp \left[- \left(\frac{x - x_0}{\eta} \right)^\beta \right]$$

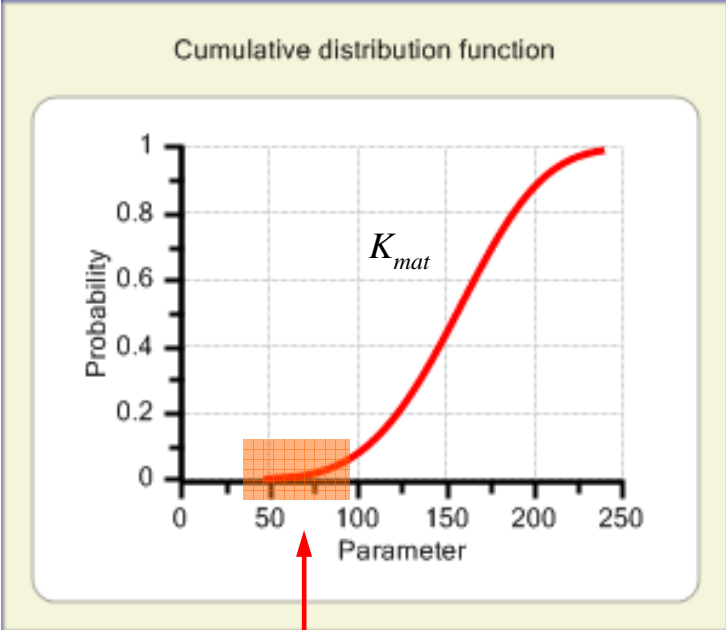
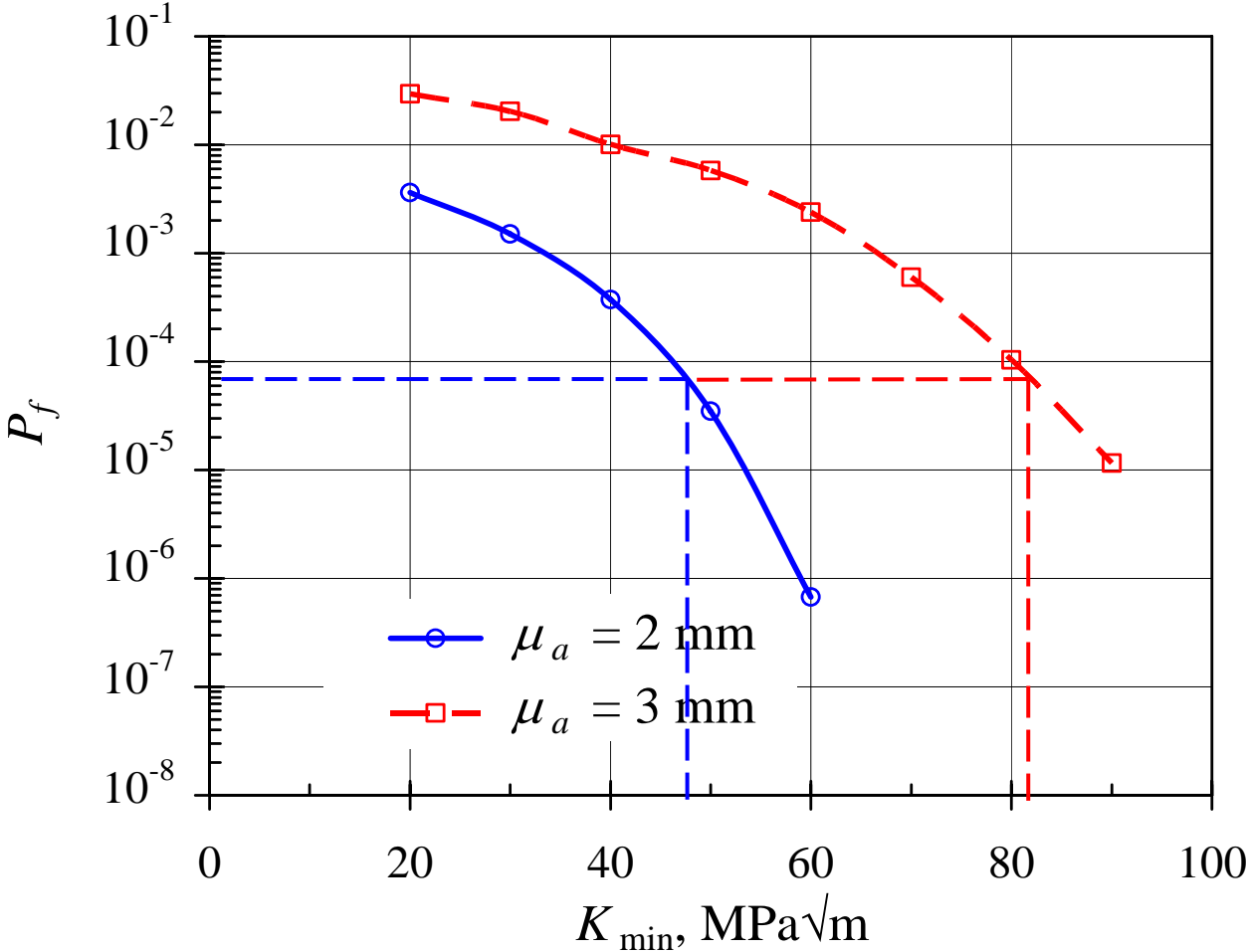
Spiral welded pipe, probabilistic assessment

Monte-Carlo simulation, $N = 10^3$: $P_f = 0.5\%$ at $\mu_a = 2$, $P_f = 2.6\%$ at $\mu_a = 3$ mm



Spiral welded pipe, probabilistic assessment

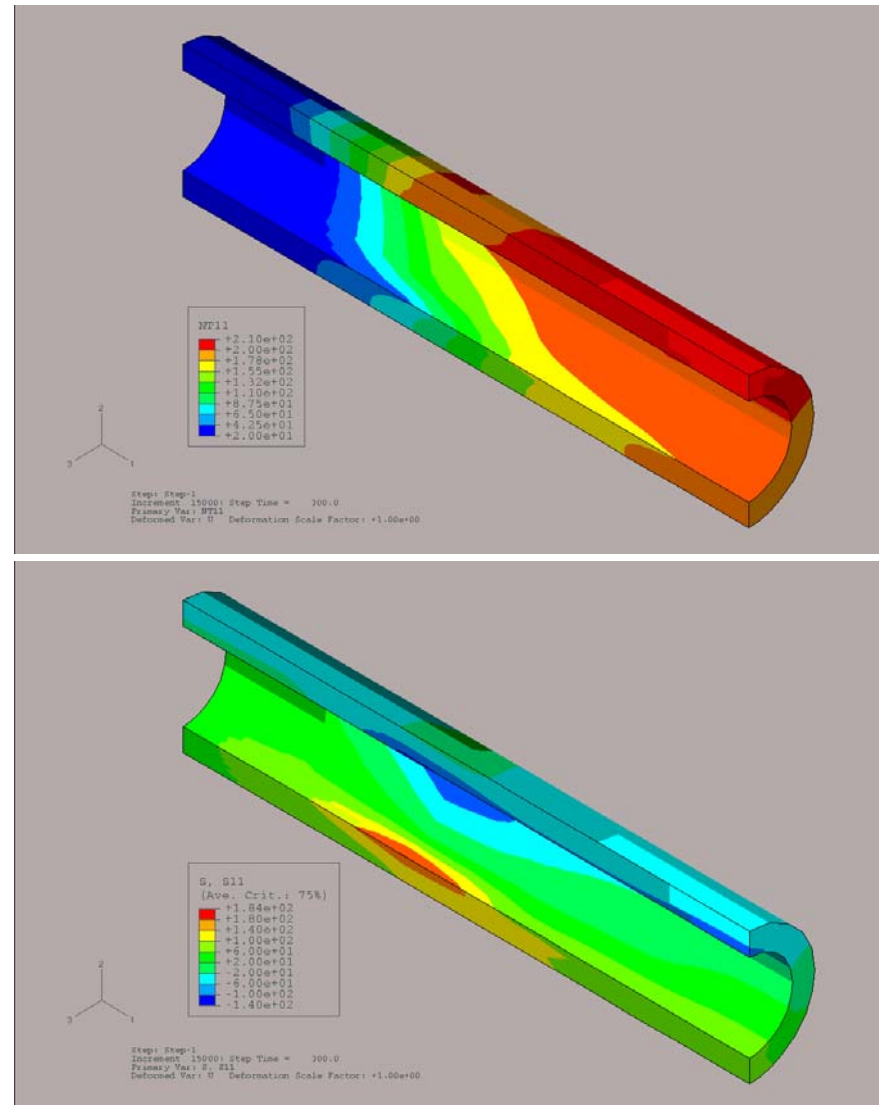
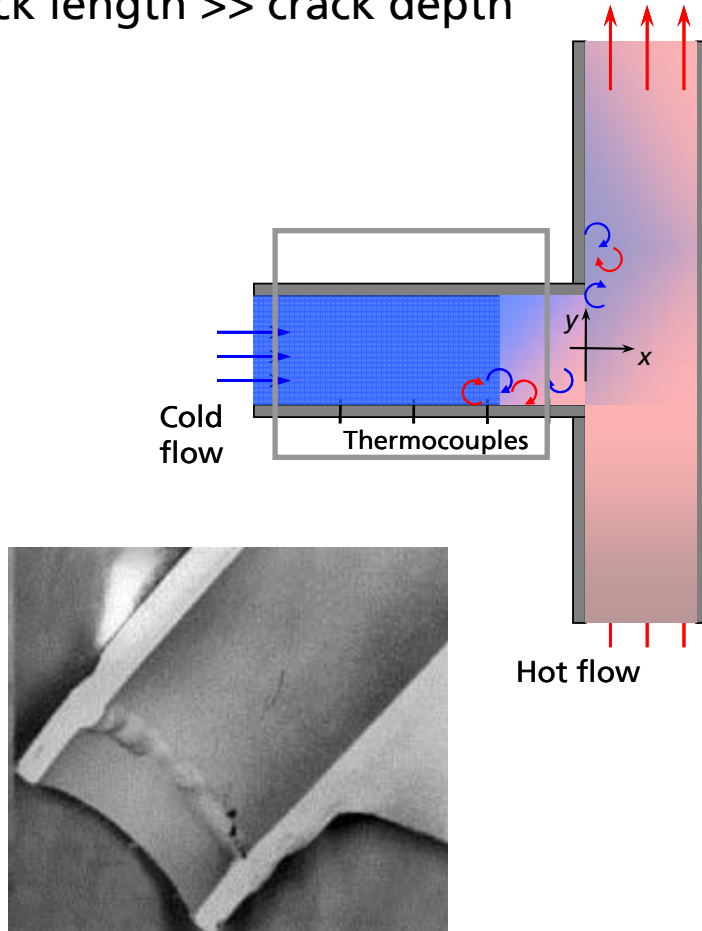
Probabilistic analysis, SORM, targeted failure probability 7×10^{-5}



Tail of distribution function

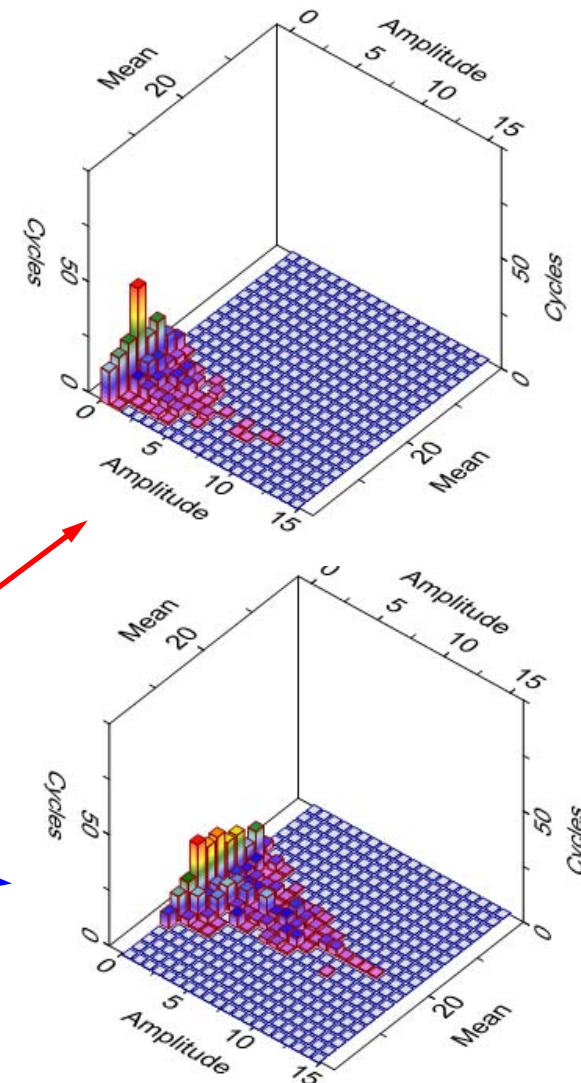
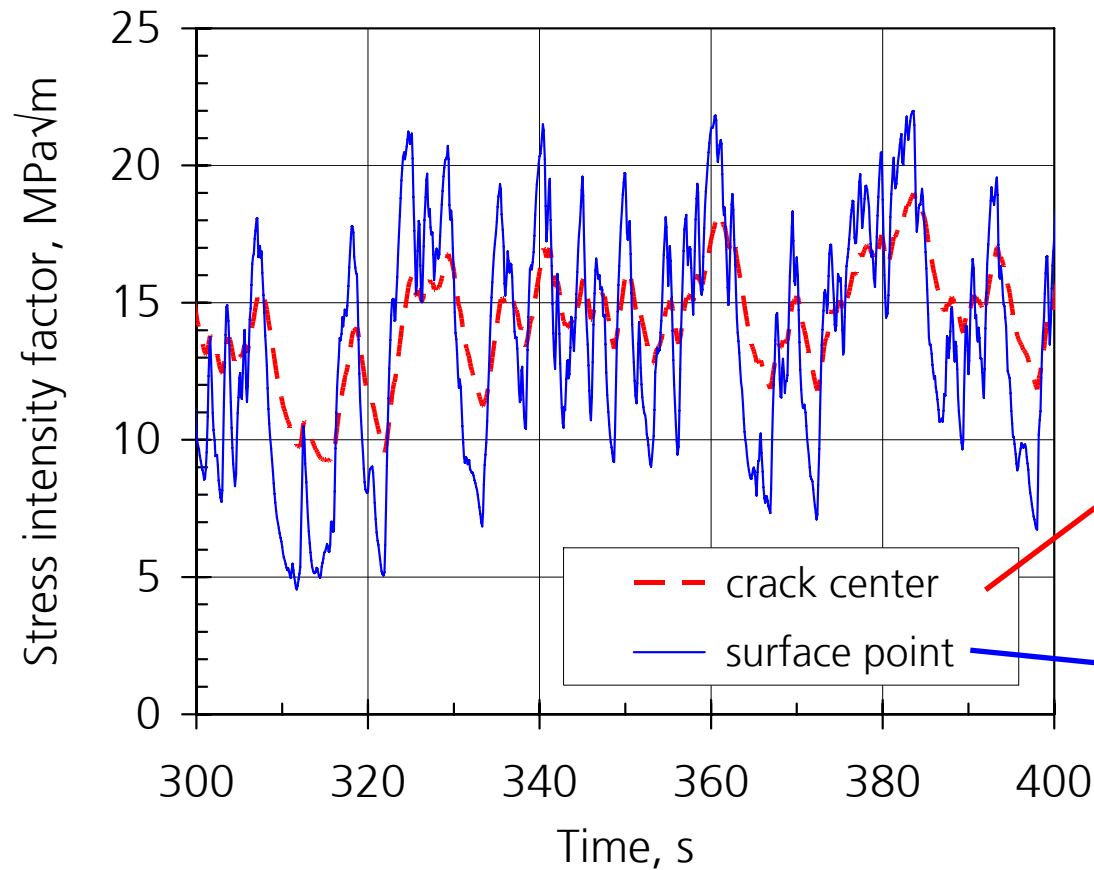
Thermal-fatigue crack growth – spectrum loading

Surface cracking in mixing zone
with crack length \gg crack depth



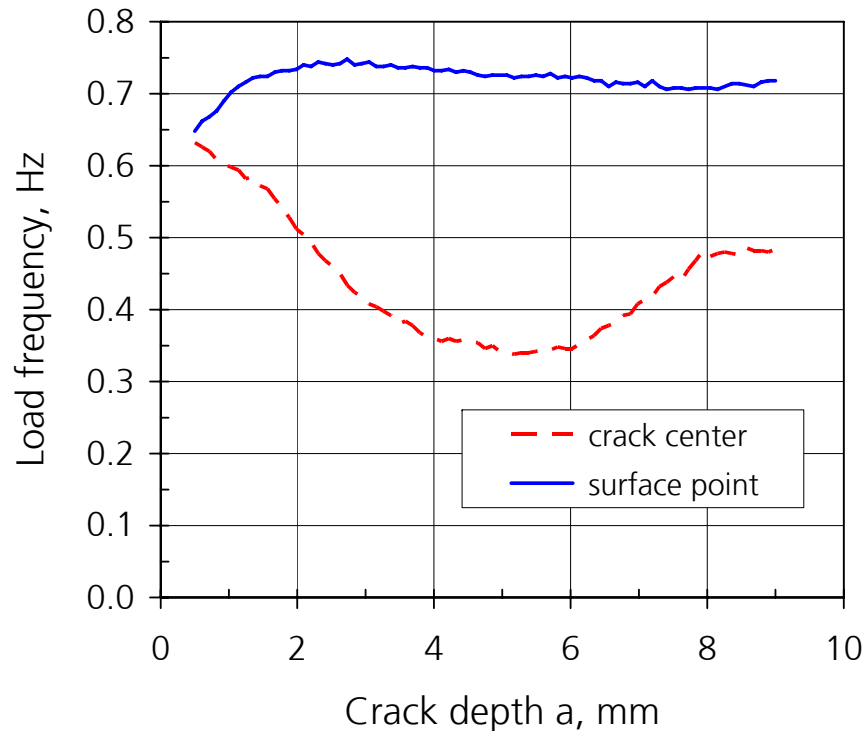
Stress intensity factor

Semi-elliptical circumferential crack



Factors influencing crack propagation

- Random loading with time dependent, 2D stress profile
- Varying load frequency along the crack front → more load cycles for the surface point
- Crack grows faster along the surface than into the wall → surface cracks with small aspect ratio a/c are predicted



Thank you for your attention!